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A linear-*rate* analog approach for the analysis of natural gas transportation networks

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A R T I C L E I N F O

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A B S T R A C T

Modeling natural gas transportation networks poses a number of challenges due to the significant nonlinearities associated to the governing equations. Pressure (nodal or *p*-) formulations and flow (nodal-loop or *q*-) formulations are the most commonly deployed approaches used to formulate gas flow network models. They treat nodal pressures and pipe branch flow rates as primary unknowns, respectively, and the Newton-Raphson method is the typical choice used to solve the resulting system of pipe network equations. The major disadvantage of Newton-Raphson methods is their tendency to hopelessly diverge when good initializations for the unknown pressure and flow variables are not available. In order to overcome this drawback, a linear-pressure analog approach, applicable to the *p*-formulation, was recently proposed to formulate an initial-guess-free solution protocol. However, solving the *q*-formulation rather than the *p*-formulation would be an alternate, practical, and desirable way to study gas flow in pipeline networks because the system of equations is predominantly linear—with the exception of the loop equations. Linear Theory and Hardy Cross methods have been used in the past solve such *q*-formulation. Unfortunately, these methods are not initial-guess-free and have been shown to become potentially unstable and thus inefficient because their convergence is also strongly associated with the availability of good initial guesses. This work proposes a *linear-rate* analog method capable of effectively solving the *q*-formulation. Through case studies, the proposed linear-rate analog method is shown to be a robust and initial-guess-free solution scheme to effectively model horizontal and inclined natural gas pipeline networks.

1. Introduction

Reliable modeling of natural gas transportation networks continues to increase in importance as a result of increased energy demands and production from large unconventional gas fields (EIA, 2013; Makogon et al., 2007; Wood et al., 2008). Solving a natural gas pipeline network system is challenging due to the high nonlinearity of the governing equations of gas flow in conduits. Therefore, the development of robust mathematical methods to model gas flow in pipeline becomes of utmost importance for the study of optimized design and performance.

Governing equations of gas pipe flow in networks are derived from energy conservation statements for steady state conditions (Ayala, 2013). Typically, pipe networks consist of nodes and pipe branches. External demands and supplies can be present at the nodes. In these networks, closed circuits formed by pipe branches are known as pipe loops. The formulation of gas flow network equations can be categorized as pressure (*p*-), pipe flow (*q*-) or loop-rate correction (Δq-) approaches, where node pressures, branch flow rates or corrections to flow rates within a loop, respectively, are treated as primary unknowns (Kelkar, 2008; Kumar, 1987). Practically, gas network formulations can be coupled with large spectrum of friction factor calculation means for various flow regimes (Chen, 1979; Colebrook, 1939; GPSA, 2004; Menon, 2005; Moody, 1944; Weymouth, 1912). The generalized Newton-Raphson method is one of the most powerful solution schemes which can be implemented to solve many types of nonlinear systems of equations. Its significant drawback is its well-known local (rather than global) convergence behavior—in which convergence is only guaranteed if the Newton Raphson scheme is initialized close-enough to the actual solution. This necessitates the availability of good initial guesses for all unknowns—which can become a daunting task for large and complex systems of equations

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and unknowns.

In order to overcome this obstacle, Ayala and Leong (2013) developed a robust and guess-free linear-pressure analog method to linearize and solve the system of equations written in terms of the *p*-formulation. The authors demonstrated that the linear-pressure analog is able to reliably converge to a final solution without the need for any pressure or flow-rate initialization. More importantly, it was shown that the analog methodology could be utilized to aid the Newton-Raphson scheme by providing very good initializations (Ayala and Leong, 2013; Leong and Ayala, 2014).

This study pursues to extend this previous work by presenting a linear-*rate* analog capable of effectively solve the *q*- (rather than the *p*-) formulation. The implementation of the *q*-formulation is practical and desirable because the resulting system of nodal mass balance equations are all linear when written in terms of flow rate—which constitute the large majority of network equations within the *q*-formulation. Other available alternatives for the solution of equations in terms of the *q*-formulation are the Hardy Cross (1930) and Linear Theory (Wood and Charles, 1972) methods. However, the implementation of the aforementioned solution techniques also requires the availability of good initial guesses. Some more recent studies have focused in speeding up the convergence of the classical methods such as Hardy Cross (e.g., Brkić, 2009) but little attention has been paid into eliminating the need for pressure or flow initial guesses from these protocols as presented in this study. In this work, the proposed linear *rate* analog linearizes the remaining non-linear loop equations—leading to a matrix system whose coefficients are fully independent from flow or pressure initializations. Detailed case studies are presented to highlight the performance of the proposed method in solving horizontal inclined network cases. Numerical performance is compared against other linearization methodologies. It is shown that the proposed method represents a robust means to generate stable and fast solutions for the *q*-formulation which could be used to provide valuable guidance during gas pipe network design and optimization.

2. Governing equations in gas network analysis

2.1. p-formulation

In the *p*-formulation, continuity equations for gas flow in conduits are written in terms of nodal pressures which are treated as primary unknowns. For a horizontal pipeline network, gas network equations are written at each node *i* considering all the connecting *j*-nodes via pipe branches as shown in Equation (1):

$$\sum_j \pm C_{ij}\sqrt{p_i^2 - p_j^2} + S_i - D_i = 0 \tag{1}$$

where C_{ij} is the pipe conductivity, and S_i and D_i are flow sources or demands at node *i*, respectively. Flows entering the node are considered positive and flows leaving the node negative. Pipe conductivity calculations involves the calculation of the appropriate friction factor depending on the gas pipe equation of interest (see Appendix A). When the *p*-formulation is applied to a network of *N* nodes, only *N-1* linearly independent equations can be obtained from Equation (1). Mathematical closure is achieved specifying pressure at one of the 'N' nodes in the system.

2.2. q-formulation

This work aims to solve governing equations for natural gas flow in conduits using the *q*-formulation. Primary unknowns in this case are flow rates at each of the pipe branches. The system of equations consists of a combination of nodal mass balance and loop equations. Nodal mass balance equations are the same as the ones presented in Equation (1), but written in terms of pipe flow rates. For an arbitrary node *i*, the nodal mass balance equation is expressed as follows:

$$\sum_j \pm q_{ij} + S_i - D_i = 0 \tag{2}$$

where q_{ij}'s are all the flows entering are leaving node *i* through pipe connections with neighboring *j*-nodes, and S_i and D_i are the external supplies and demands at node *i*, respectively. Flows entering the node are considered positive and flows leaving the node negative. Equation (2) is analogous to the first law of Kirchhoff for conservation of electrical charge in electrical circuits. In a pipe network of with *N* nodes, *N-1* linear dependent equations can be generated via Equation (2). If "*L*" loops can be identified in the network system, additional loop equations can be established for each loop via the second law of Kirchhoff. Around every closed loop, the sum of all pressure drops around the geometrical enclosure must add up to zero:

$$\sum_{i,j}^{loop} \pm\left(p_i^2 - p_j^2\right) = 0 \tag{3}$$

In this summation (Equation (3)), pressure drops that result in flow directions that align with prescribed loop direction are considered positive, and negative otherwise. According to the generalized gas pipe equation for the horizontal case (see Appendix A), one writes:

$$\left(p_i^2 - p_j^2\right) = \left(\frac{q_{ij}}{C_{ij}}\right)^2 = R_{ij}\, q_{ij}^2 \tag{4}$$

where R_{ij} is the pipe resistance ($R_{ij} = 1/C_{ij}^2$). Substituting Equation (4) into Equation (3), loops equations could be alternatively recast as:

$$\sum_{ij}^{loop} \pm R_{ij}\, q_{ij}^2 = 0 \tag{5}$$

It should be reiterated at this point that the resulting system of equations generated using the *q*-formulation is mostly linear: "*N-1*" equations are already linear in flow rate (nodal mass conservation, Equation (2)) with additional "*L*" non-linear equations (loop equations, Equation (5)) because of the presence of squared flow (q_{ij}^2) terms. In general, N >> L, and the system of equations would be fully linear if there were no closed loops in the network (L = 0).

3. The linear-*pressure* analog formulation

Ayala and Leong (2013) developed a linear-pressure analog scheme that linearized the *p*-form of the gas network equations. They showed that the non-linear pressure coefficient in the generalized gas flow equation (Equation (6)):

$$q_{ij} = C_{ij}\sqrt{p_i^2 - p_j^2} \tag{6}$$

can be rewritten as Equation (7):

$$\sqrt{p_i^2 - p_j^2} = \sqrt{\frac{p_i + p_j}{p_i - p_j}}\left(p_i - p_j\right) = \sqrt{1 + \frac{2}{\frac{p_i}{p_j} - 1}}\cdot\left(p_i - p_j\right) \tag{7}$$