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Mode pair selection of circumferential guided waves for cumulative second-harmonic generation in a circular tube

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The appropriate mode pairs of primary and double-frequency circumferential guided waves (CGWs) have been investigated and selected for generation of the cumulative second harmonics, which are applicable for quantitative assessment of damage/degradation in a circular tube. The selection criteria follow the requirements: the higher efficiency of cumulative second-harmonic generation (SHG) of primary CGW propagation, and the larger response sensitivity of cumulative SHG to material damage/degradation [characterized by variation in the third-order elastic (TOE) constants]. The acoustic nonlinearity parameter β of CGW propagation and the change rate of normalized β versus the TOE constants of tube material are, respectively, used to describe the efficiency of SHG and its response sensitivity to damage/degradation. Based on the selection criteria proposed, all the possible mode pairs of primary and double-frequency CGWs satisfying the phase velocity matching have been numerically examined. It has been found that there are indeed some mode pairs of CGW propagation with the larger values both of β and the change rate of normalized β versus the TOE constants. The CGW mode pairs found in this paper are of practical significance for quantitative assessment of damage/degradation in the circular tube.

1. Introduction

It is known that nonlinear ultrasonic technique has the potential for accurate assessment of structural integrity owing to its enhanced sensitivity to the level of damage/degradation as compared to conventional linear ultrasonic one [1–4]. Especially, in recent years, the effect of the second-harmonic generation (SHG) by primary (fundamental frequency) ultrasonic guided wave propagation in structural components has been drawn much attention and extensively studied for accurately assessing the changing aspects of material microstructures, such as those caused by plastic strains, fatigue, creep damage, thermal degradation, and other types of microdamages [5].

Generally, the effect of SHG of ultrasonic guided wave propagation is inefficient and easily overlooked due to its dispersive nature. In order to achieve an effective generation of second harmonics, Deng first found that the second harmonics generated could grow linearly with propagation distance once primary and double-frequency guided wave modes satisfied the phase velocity matching condition [6,7]. Subsequently, a series of theoretical and experimental investigations have been conducted on the nonlinear ultrasonic guided waves propagating in plates, which further reveal the physical mechanism of generation of cumulative second harmonics with respect to the mode pairs satisfying the phase velocity matching and nonzero energy transfer from primary to double-frequency guided wave mode (i.e., nonzero energy flux) [8–13]. Based on the researches of SHG of ultrasonic guided waves in elastic plates, Müller et al. and Matlack et al., respectively, conducted the investigations for the ratio of accumulation of second harmonics of different Lamb wave modes through theoretical analyses and experimental measurements [14,15]. Matsuda et al. further analyzed features of specific Lamb wave mode pairs satisfying phase and group velocity matching for cumulative SHG of primary Lamb wave propagation [16], and Liu et al. investigated selection of primary guided wave modes for generation of strong internally resonant second harmonics in plate based on requirements of phase and group velocity matching, nonzero power flux, and modal excitability/receivability [17]. Results of those investigations performed provide some appropriate mode pairs of primary and double-frequency guided waves in plates, through which an effective generation and detection of second harmonics can be implemented. It is worth noting that previous works [14–17] have only focused on the efficiency of strong cumulative SHG for quantitatively assessing the changing aspects of material

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microstructures, while the response sensitivity of cumulative SHG to the level of damage/degradation in material has rarely been investigated simultaneously.

The performed investigations on the effect of cumulative SHG and its applications mainly focus on the cases where ultrasonic guided waves propagate in plate-like structures [6–20], or propagate axially in arbitrary constant cross-section waveguides such as cylinders/rods [21–25]. As a kind of elementary guided wave mode propagating along the circumference of a circular tube [referred to as circumferential guided wave (CGW) mode] [26,27], investigations on its nonlinear effect have recently drawn much attention [28–31]. It has been experimentally verified that early damage in tube material can be quantitatively assessed using the acoustic nonlinearity parameter as measured with the second harmonics generated by primary CGW propagation [31]. Theoretically, for effectively and accurately assessing material damage/degradation in a circular tube, it is necessary to select appropriate mode pairs of primary and double-frequency CGWs with the higher efficiency of cumulative SHG and the larger sensitivity of its response to material damage/degradation. However, investigation on selection of CGW mode pairs capable of satisfying the abovementioned requirements has not been conducted.

For the purpose of effectively and accurately assessing damage/degradation in tube material, this study reports an investigation focusing on mode pair selection of CGW propagation for generation of cumulative second harmonics in a circular tube. The selection criteria follow the requirements: the higher efficiency of cumulative SHG and the larger sensitivity of its response to material damage/degradation (characterized by variation in the third-order elastic (TOE) constants [1]). The acoustic nonlinearity parameter β of primary CGW propagation and the change rate of normalized β versus the TOE constants of the tube material are, respectively, used to describe the efficiency of SHG and its response sensitivity to damage/degradation. It has been found that there are indeed some appropriate mode pairs of CGW propagation with the larger values both of β and the change rate of normalized β versus the TOE constants. The mode pairs found in this study are of practical significance for quantitative assessment of damage/degradation in the circular tube. Of course, the selection criteria of mode pairs proposed in this study are applicable to other types of guided waves.

2. Theoretical fundamentals

The schematic of a two-dimensional model of the circular tube for analyzing CGW propagation is illustrated in Fig. 1, where the

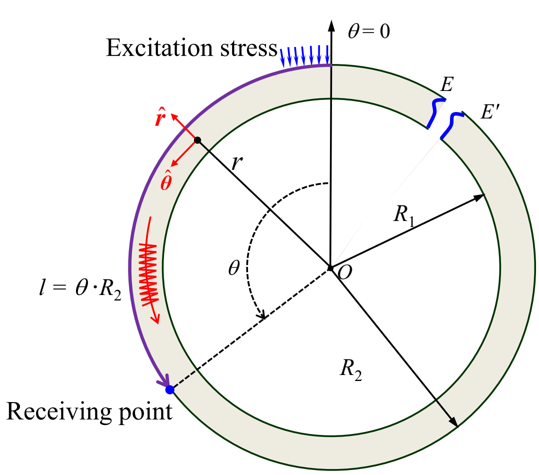

Fig. 1. Schematic of a two-dimensional model for analyzing CGW propagation in a circular tube.

Lagrangian cylindrical coordinates (r, θ) are established. By means of the equations of stress-free boundary conditions at the inner and outer surfaces of the circular tube, both the dispersion relations for CGW propagation and the corresponding displacement fields can readily be determined [26–28,31]. When a primary CGW mode (with the angular frequency ω, $\omega = 2\pi f$) propagates counterclockwise along the tube circumference shown in Fig. 1, the corresponding mechanical displacement field $\mathbf{U}^{(\omega)}$ can formally be written as [31]

$$\mathbf{U}^{(\omega)} = \mathbf{U}^{(\omega)}(r)\exp[\mathrm{j}n^{(\omega)}\theta - \mathrm{j}\omega t] \tag{1}$$

where $\mathbf{U}^{(\omega)}(r)$ is the field function of primary CGW, and $n^{(\omega)} = \omega R_2/c_{\mathrm{p}}^{(\omega)}$ and $c_{\mathrm{p}}^{(\omega)}$ are, respectively, the dimensionless angular wave number and the phase velocity of the primary CGW mode. The components of $\mathbf{U}^{(\omega)}$ along the radial and circumferential directions are given by $U_r = \hat{\mathbf{r}} \cdot \mathbf{U}^{(\omega)}$ and $U_\theta = \hat{\boldsymbol{\theta}} \cdot \mathbf{U}^{(\omega)}$, where $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are, respectively, the unit radial and circumferential vectors. Within a second-order perturbation, when the primary CGW mode propagates along the tube circumference, the bulk driving force $\mathbf{F}^{(2\omega)}$ of double the fundamental frequency in the interior of the circular tube shown in Fig. 1, as well as the traction stress tensor $\mathbf{P}^{(2\omega)}$ of double the fundamental frequency (i.e. the second-order term of the first Piola-Kirchhoff stress tensor [32]) on the inner and outer surfaces of the circular tube, can be generated due to the convective nonlinearity and the inherent elastic nonlinearity of solid [8,9,28,31,32]. The components of the bulk driving force $\mathbf{F}^{(2\omega)}$, denoted by $F_r^{(2\omega)} = \hat{\mathbf{r}} \cdot \mathbf{F}^{(2\omega)}$ and $F_\theta^{(2\omega)} = \hat{\boldsymbol{\theta}} \cdot \mathbf{F}^{(2\omega)}$, are, respectively, given by [28,31]

$$F_r^{(2\omega)} = \frac{1}{r}\frac{\partial(rP_{rr}^{(2\omega)})}{\partial r} + \frac{1}{r}\frac{\partial P_{r\theta}^{(2\omega)}}{\partial \theta} - \frac{P_{\theta\theta}^{(2\omega)}}{r} \tag{2}$$

$$F_\theta^{(2\omega)} = \frac{1}{r}\frac{\partial(rP_{rr}^{(2\omega)})}{\partial r} + \frac{1}{r}\frac{\partial P_{r\theta}^{(2\omega)}}{\partial \theta} - \frac{P_{\theta\theta}^{(2\omega)}}{r} \tag{3}$$

where $P_{rr}^{(2\omega)} = \hat{\mathbf{r}} \cdot \mathbf{P}^{(2\omega)} \cdot \hat{\mathbf{r}}, P_{r\theta}^{(2\omega)} = \hat{\mathbf{r}} \cdot \mathbf{P}^{(2\omega)} \cdot \hat{\boldsymbol{\theta}}$, and $P_{\theta\theta}^{(2\omega)} = \hat{\boldsymbol{\theta}} \cdot \mathbf{P}^{(2\omega)} \cdot \hat{\boldsymbol{\theta}}$ are, respectively, the components of $\mathbf{P}^{(2\omega)}$, and its formal expressions related to the second-order elastic constants (λ, μ) and the TOE ones (A, B, and C) are given in Refs. [28,31].

According to the modal expansion approach for waveguide excitation [8,9,28,31], $\mathbf{F}^{(2\omega)}$ and $\mathbf{P}^{(2\omega)}$ can respectively be assumed to be the bulk and surface sources for generation of a series of double-frequency CGW modes that constitute the second-harmonic field [denoted by $\mathbf{U}^{(2\omega)}(r,\theta)$] of the primary CGW propagation, namely, $\mathbf{U}^{(2\omega)}(r,\theta) = \sum_m A_m(\theta) \times \mathbf{U}^{(2\omega,m)}(r)$, where $A_m(\theta)$ is the expansion coefficient of the field function [denoted by $\mathbf{U}^{(2\omega,m)}(r)$] of the mth double-frequency CGW mode. Based on the reciprocity relation and the orthogonality of guided wave modes, the equation governing the expansion coefficient $A_m(\theta)$ can be obtained, and further the solution to $A_m(\theta)$ can formally be given by [28,31]

$$\begin{aligned} A_m(\theta) &= A_m \sin(\Delta n\theta)/\Delta n \times \exp(\mathrm{j}\Delta n\theta) \times \exp[\mathrm{j}n^{(2\omega,m)}\theta] \\ A_m &= [F_m^{\mathrm{S}} + F_m^{\mathrm{V}}]/4P_{mm} \end{aligned}, \tag{4}$$

where F_m^{V} (originated from $\mathbf{F}^{(2\omega)}$), F_m^{S} (originated from $\mathbf{P}^{(2\omega)}$) and P_{mm} are, respectively, the bulk source, surface source and the average power flow (per unit width, perpendicular to the tube section) of the mth double-frequency CGW mode, whose formal expressions are also given in Refs. [28,31]. $n^{(2\omega,m)}$ is the dimensionless angular wave number of the mth double-frequency CGW, $\Delta n = [n^{(\omega)} - n^{(2\omega,m)}/2]$ is used to describe the degree of dispersion (i.e., the difference of phase velocity between the primary and the mth double-frequency CGWs). It is found that $A_m(\theta)$ in Eq. (4) can grow linearly with the circumferential angle θ when $A_m \neq 0$ and $\Delta n = 0$