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Wave propagation through a flexoelectric piezoelectric slab sandwiched by two piezoelectric half-spaces

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ABSTRACT

Reflection and transmission of plane waves through a flexoelectric piezoelectric slab sandwiched by two piezoelectric half-spaces are studied in this paper. The secular equations in the flexoelectric piezoelectric material are first derived from the general governing equation. Different from the classical piezoelectric medium, there are five kinds of coupled elastic waves in the piezoelectric material with the microstructure effects taken into consideration. The state vectors are obtained by the summation of contributions from all possible partial waves. The state transfer equation of flexoelectric piezoelectric slab is derived from the motion equation by the reduction of order, and the transfer matrix of flexoelectric piezoelectric slab is obtained by solving the state transfer equation. By using the continuous conditions at the interface and the approach of partition matrix, we get the resultant algebraic equations in term of the transfer matrix from which the reflection and transmission coefficients can be calculated. The amplitude ratios and further the energy flux ratios of various waves are evaluated numerically. The numerical results are shown graphically and are validated by the energy conservation law. Based on these numerical results, the influences of two characteristic lengths of microstructure and the flexoelectric coefficients on the wave propagation are discussed.

1. Introduction

It is known that the classical elastic theory does not suffice for an accurate and detailed description of corresponding mechanical behavior in the range of micro and nano scales. The main cause is the absence of internal characteristic length, characteristic of the underlying microstructure, from the constitutive equation in the classical elastic theory, and therefore the notable size effects observed experimentally could not be captured. In the problem of wave propagation, the classical elastic theory is also believed to be inadequate for a material possessing microstructure, in particular, when the wavelength of an incident wave is comparable to the length of the material microstructure. The same, in the classical piezoelectric elastic theory, no characteristic length is included in the constitutive relations. Therefore, the classical piezoelectric elastic theory cannot describe the mechanical and electrical behaviors of piezoelectric material in the micro or nano scale and size effects. Recently, Zubko et al. [1] studied the flexoelectric effect in piezoelectric solids. They discussed the presence of flexoelectric effect in many nanoscale systems and looked at its potential applications in the MEMS (micro electro mechanical system). Hu and Shen [2,3] studied the variational principles and governing equations in nano-dielectrics with the flexoelectric effect. By establishing the electric enthalpy variational principle for nano-sized dielectrics with the strain gradient and the polarization gradient effects, the governing equations and boundary conditions were given. Shu et al. [4] studied the symmetry of flexoelectric coefficients in crystalline medium. Their investigation indicated that the direct flexoelectric coefficients should be presented in 3×18 form and the converse flexoelectric coefficients in 6×9 form, rather than 6×6 form. Liang et al. [5] studied the Bernoulli–Euler dielectric beam model based on the strain-gradient effect. It was found that the beam deflection predicted by the strain gradient beam theory is smaller than that by the classical beam theory when the beam thickness is comparable to the internal length scale parameters. Zhang et al. [6] demonstrated an experiment on two designs of flexoelectric metamaterials. It was found that when a ferroelectric ceramic wafer is placed on a metal ring or has a domed shape, which is produced through the diffusion between two pieces of ferroelectric ceramic of different

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compositions at high temperatures, an apparent piezoelectric response originating from the flexoelectric effect can be measured under a stress, and the apparent piezoelectric response of the materials based on the designs can be sustained well above the Curie temperature. Liu and Wang [7] investigated the size-dependent electromechanical properties of piezoelectric superlattices made of $BaTiO_3$ and $PbTiO_3$ layers, it was found that the strain gradient is giant at the interface between $BaTiO_3$ and $PbTiO_3$ layers, which will lead to the significant enhancement of polarization in the superlattice due to the flexoelectric effect, therefore, the influence of strain gradient at the interface becomes significant when the size of superlattice decreases. Although the flexoelectric effect has been studied in the above-mentioned literatures, the influences of the flexoelectric effect on the wave propagation in piezoelectric material have not been reported so far.

The reflection and transmission of elastic wave through a slab with finite thickness was an everlasting interesting topic in the past decades. Caviglia and Morro [8,9] studied the wave propagation through an elastic slab and a viscoelastic slab, respectively. Tolokonnikov [10] studied the wave propagation through an inhomogeneous anisotropic slab. Larin and Tolokonnikov [11] further studied the wave propagation through a non-uniform thermoelastic slab. Hsia and Su [12] studied the wave propagation through a microporous slab characteristic of micropolar elasticity. Zhang et al. [13] also studied the wave propagation through a micropolar slab sandwiched by two elastic half-spaces. The influences of the micropolar elastic constants and the thickness of slab on the reflection and transmission waves were discussed in their paper. Li and Wei [14,15] studied the reflection and transmission of plane waves at the interface between two different dipolar gradient elastic half-spaces and the reflection and transmission through a microstructured slab sandwiched by two half-spaces. It was found that the microstructure effects make the propagating waves dispersive and create the evanescent waves that become the surface waves at the interface. The influences of three characteristic lengths, namely, the incident wavelength, the thickness of slab and the characteristic length of microstructure, on the reflection and transmission waves were analyzed. Because the sandwiched structure is widely met in the transducer, actuator, acoustic isolator, interface detection and so on, the researches on the wave propagation through a sandwiched slab are of importance theoretically and practically.

In this paper, wave propagation through a piezoelectric slab sandwiched by two piezoelectric half-spaces with the flexoelectric effect taken into consideration is studied. The dispersive equation is derived from the general motion equation and the all possible partial waves in the piezoelectric material with the flexoelectric effect are discussed. The state vectors and the state transfer equation in the piezoelectric slab are also derived. Non-traditional interface conditions with the monopolar and dipolar tractions are considered, and the reflection and transmission coefficients are obtained in term of the transfer matrix. The amplitude ratios and further the energy flux ratios of the reflection and transmission waves are calculated numerically and the numerical results are validated by the check of energy conservation. Based on the numerical results, the influences of the flexoelectric coefficients and two characteristic lengths of microstructure on the reflection and transmission waves are discussed.

2. State vectors in the piezoelectric solid with the flexoelectric effect considered

The general expression of the electric Gibbs free energy density function can be written as [5]

$$U = -\frac{1}{2}\varepsilon_{kl}E_kE_l + \frac{1}{2}c_{ijkl}S_{ij}S_{kl} - e_{kij}E_kS_{ij} - f_{ijkl}E_i\eta_{jkl} + r_{ijklm}S_{ij}\eta_{klm} + \frac{1}{2}g_{ijklmn}\eta_{ijk}\eta_{lmn}, \tag{1}$$

where ε_{kl} and c_{ijkl} are the dielectric and elastic tensors, respectively. e_{kij} is the piezoelectric tensor, f_{ijkl} is flexoelectric coefficient tensor, r_{ijklm} denotes the coupling between the strain and strain gradient. The tensor g_{ijklmn} represents the strain gradient effect. S_{ij} is the strain tensor, η_{jkl} is the strain gradient tensor and E_i is the electric field vector, which are defined, respectively, as $S_{ij} = (u_{j,i} + u_{i,j})/2$, $\eta_{ijk} = S_{ij,k}$ and $E_i = -\varphi_{,i}$, where u is the displacement, φ is the electric potential, the comma indicates differentiation with respect to the spatial variables.

It is noted that

$$\int_V r_{ijklm}S_{ij}\eta_{klm}dV = \frac{1}{2}\int_V r_{ijklm}(S_{ij}S_{kl})_{,m}dV = \frac{1}{2}\int_S (S_{ij}S_{kl})r_{ijklm}n_m dS. \quad (r_{ijklm} = r_{klijm}) \tag{2}$$

This means that the fifth term in the right side of Eq. (1) is the contribution from the surface of material. Let

$$U^s = \frac{1}{2}r_{ijklm}S_{ij}\eta_{klm}, \tag{3}$$

then, U^s is the surface energy of unit surface area and it includes the contribution from the surface stresses. In the present work, the surface effects of material are neglected, namely, r_{ijklm} is null. Moreover, g_{jkhmni} is approximated by $g_{jkhmni} = l_1^2\delta_{hi}c_{jkmn}$ [16], where l_1 is internal characteristic length of microstructure. Then, the constitutive equations can be obtained from the electric Gibbs free energy as

$$\sigma_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k, \tag{4a}$$

$$\sigma_{jkh} = -f_{ijkh}E_i + l_1^2\delta_{hi}c_{jkmn}\eta_{mni}, \tag{4b}$$

$$D_k^e = \varepsilon_{kl}E_l + e_{kij}S_{ij} + f_{klmn}\eta_{lmn}, \tag{4c}$$

where σ_{ij} is the classical Cauchy stress tensor, σ_{jkh} is the higher-order stress tensor, and D_k^e is the electric displacement vector. It is noted that $\sigma_{ij} = \sigma_{ji}$ and $\sigma_{jkh} = \sigma_{kjh}$.

The kinetic energy density with consideration of the micro-inertial effect can be expressed as [14,15]

$$T = \frac{1}{2}\rho\dot{u}_j\dot{u}_j + \frac{1}{6}\rho l_2^2\dot{u}_{k,j}\dot{u}_{k,j}, \tag{5}$$

where ρ is mass density, and l_2 is the micro inertia characteristic length. Then, the variation of the electric Gibbs free energy density (electrostatic force is neglected) plus the kinetic energy density are [2,3,14],

$$\delta\int_V (U+T)dV = \int_a \left[(\sigma_{ij} - \sigma_{ijm,m})n_j + D_l(n_l)\sigma_{ijm}n_jn_m - D_j(\sigma_{ijm}n_m) + \frac{\rho l_2^2}{3}n_j\ddot{u}_{i,j}\right]\delta u_i da - \int_V D^e_{m,m}\delta\varphi dV - \int_V [(\sigma_{ij} - \sigma_{ijm,m})_{,j} - (\rho\ddot{u}_i - \frac{\rho l_2^2}{3}\ddot{u}_{i,jj})]\delta u_i dV + \int_a \sigma_{ijm}n_jn_m D\delta u_i da + \int_a n_iD_i^e\delta\varphi da, \tag{6}$$

where n_j is the unit normal vector of the boundary of solid, $D_j(\,) = (\,)_{,j} - n_jn_k(\,)_{,k}$, $D(\,) = n_l(\,)_{,l}$.

The variation of work done by external forces can be expressed as

$$\delta W = \int_V F_i\delta u_i dV + \int_a P_i\delta u_i da + \int_a R_iD\delta u_i da + \int_a q\delta\varphi da, \tag{7}$$