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Short communication

Physical constraints on the non-dimensional absorption coefficients of compressional and shear waves for viscoelastic cylinders

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ABSTRACT

Background: Normalized absorption coefficients for the longitudinal and shear waves in viscoelastic (polymer-type) materials, extracted from non-fictional experimental data showed anomalous effects, such as the generation of a negative radiation force (NRF) in plane progressive waves, negative energy absorption and extinction efficiencies and a scattering enhancement, not in agreement with energy conservation.
Objective: The objective of this work is directed towards analyzing those anomalies from the standpoint of energy conservation. Physical conditions which demonstrate that the ratio of the normalized absorption coefficients cannot be of arbitrary value but depends on the ratio of the square of the compressional and shear wave speeds, are established and discussed.
Method: The necessary physical condition for the validity of the linear viscoelastic (VE) model for any passive (i.e. that does not generate energy) polymeric cylinder with an ultrasonic absorption of hysteresis-type submerged in a non-viscous fluid requires that the absorption efficiency be positive ($Q_{abs} > 0$) since there are no active radiating sources inside the core material. This condition imposes restrictions on the values attributed to the normalized absorption coefficients for the compressional and shear-wave wavenumbers for each partial-wave mode n. The forbidden values produce anomalous/unphysical NRF, negative absorption and extinction efficiencies, as well as an enhancement of the scattering efficiency using plane progressive waves, not in agreement with energy conservation.
Results: Based on the partial wave series expansion method in cylindrical coordinates, numerical results for the radiation force, extinction, absorption and scattering energy efficiencies assuming plane progressive wave incidence are performed for three VE polymer cylinders immersed in a non-viscous host liquid (i.e. water) with particular emphasis on the shear-wave absorption coefficient, the dimensionless size parameter ka (where k is the wavenumber and a is the radius of the cylinder) and the partial-wave mode number n. Physical and mathematical conditions are established for the non-dimensional absorption coefficients of the longitudinal and shear waves for a cylinder (i.e. the 2D case) in terms of the sound velocities in the VE material. The physical condition for the spherical 3D case is also noted.
Conclusion: For passive materials, the physical conditions must be always satisfied to allow accurate computations of the acoustic radiation force, torque, and energy absorption, extinction and scattering efficiencies for VE cylinders having a hysteresis type of absorption (such as polymers and plastics), and submerged in a non-viscous fluid. The physical conditions must be always satisfied regardless of the shape of the incident field. They also serve to validate and verify experimental data for VE materials and test the accuracy of related numerical computations.

1. Introduction

Mathematical modeling of absorption and scattering by polymer viscoelastic (VE) materials subjected to acoustical waves [1] is an active field of research in various fields. Such materials are used, for example, in the coating of gas-filled microbubbles [2] in

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biomedical ultrasound imaging applications using contrast agents, and drug delivery [3–6]. The use of polymers [such as Polyethylene (PE) and Phenolic Polymer (PP)] has been also suggested in the design of single- and double-layered spherical shells for potential encapsulated drug delivery [7]. Other areas have included the fabrication of elongated bone samples based on polymer composite materials, such as ebonite, acrylic plastic, fiberglass, and carbon fiber plastic [8]. An additional area of interest concerns the fabrication of polymer-based tubular channels [made of a mixture of silica, cellulose and polymethylmethacrylate (PMMA)] used as blood vessel phantoms in *in vitro* studies for the characterization of contrast agents subjected to ultrasound [9]. Polymers can be also used in the design and manufacture of acoustic filters and mufflers for noise control [10], where the nondestructive evaluation by resonance acoustic spectroscopy of tubular cylinders [11,12] is applied for structural health monitoring purposes.

For a mathematical model describing the behavior of an isotropic VE polymer-type material in which the stress varies linearly with strain, a hysteresis-type of absorption has been assumed and validated experimentally [13]. Examples for such materials include PMMA, PE and PP [13,14]. For such materials, absorption varies linearly with frequency, thus, the attenuation of compressional and shear-waves inside the material core can be modeled by introducing complex wavenumbers [15] into the acoustic scattering theory. Therefore, the expressions for the complex wavenumbers corresponding to the compressional and shear-waves depend on normalized absorption coefficients independent of frequency [16,17], which account for the attenuation of acoustical waves within the material of the scattering object.

This linear VE model assuming a hysteresis-type of absorption in plastic polymers has been utilized in various investigations dealing with numerical computations for the acoustic scattering [18–21], and radiation forces on a sphere or cylinder [22–27] in the development of improved acoustical tweezers devices [28–32], that can be coated by a layer of a VE material [33–37] in a non-viscous fluid. Moreover, the acoustic radiation force and torque experienced by a VE polymer sphere or spherical shell filled with water or air in the field of a Bessel (vortex) beam have been evaluated, based on this VE model [38–40].

Although numerical computations [excluding the long-wavelength (Rayleigh) limit] for the acoustic backscattering form function (i.e. scattered pressure in the far-field) from a VE PMMA sphere immersed in a non-viscous fluid obtained using this VE model correlate with measurements of the normalized backscattering pressure [20], its domain of validity has not been examined. Moreover, numerical predictions for the acoustic radiation force and energy efficiency factors using normalized (i.e. dimensionless) absorption coefficients for the compressional γ_L and shear-waves γ_S extracted from non-fictional experimental data (for high-density PE) [41] using the same mathematical VE model have shown anomalous effects for a certain range of values and choice of parameters (discussed in the following). Those inconsistencies in the experimental data [41] have been also noted in [42].

The purpose of this work is to investigate related effects from the standpoint of the acoustic radiation force and acoustical energy efficiencies for a VE cylinder submerged in non-viscous water. The validity of the VE model for circular cylinders is reexamined via the optical theorem (known otherwise as the extinction or cross-section theorem) for acoustical beams in cylindrical coordinates [43] used here for acoustical plane waves. The derived formalism and physical conditions are based upon the law of the conservation of energy for arbitrary objects in scalar acoustical beams of arbitrary shape [43–45]. Since the physical phenomena involve the interaction of acoustical waves with a passive object, i.e. one with no active sources of energy present in its interior, the energy should decrease by virtue of the passive absorption properties (i.e. viscous losses) of VE materials (knowing as passivity [46]), leading to positive absorption efficiency (or power). This condition stemming from first principles leads to physical restrictions on the normalized absorption coefficients for the compressional and shear waves as will be shown subsequently. The analysis here is focused on the case of a VE cylinder, though the case of a VE sphere [42,47] has been recently considered. In Section 2, the series expansion in cylindrical multipole partial-waves is used, which allows adequate analytical and computational analyses of the radiation force and energy efficiency factors. The results are illustrated and discussed, and the physical conditions are established. Section 3 provides the conclusion of this work.

2. Method, numerical results and discussions

The interaction of acoustical waves with a cylinder involves the introduction of various concepts used to explain physical phenomena, such as the generation of the radiation force [48,49], the acoustic extinction, and scattering in the host fluid [50,51]. The analysis is started by considering the analytical expression for the radiation force function Y_p of plane progressive waves incident upon a VE cylinder at normal incidence with respect to its geometrical axis as [25,52],

$$Y_p = -\frac{2}{ka}\sum_{n=0}^{\infty}\left[\alpha_n + \alpha_{n+1} + 2(\alpha_n\alpha_{n+1} + \beta_n\beta_{n+1})\right], \tag{1}$$

where the coefficients $\alpha_n = \mathrm{Re}(C_n)$ and $\beta_n = \mathrm{Im}(C_n)$ are the real and imaginary parts of the scattering coefficients C_n used in the description of the scattered field from a cylinder (p. 348 in [53]). The parameter k is the wavenumber and a is the cylinder radius.

Moreover, the expressions for the efficiency factors, representing the relative amount of the energy that is extinguished, absorbed and scattered by the VE cylinder are considered. They are expressed, respectively, as [43],

$$Q_{\mathrm{ext}} = -\frac{2}{ka}\sum_{n=0}^{\infty}\varepsilon_n\alpha_n, \tag{2}$$

$$Q_{\mathrm{abs}} = -\frac{2}{ka}\sum_{n=0}^{\infty}\varepsilon_n\left(\alpha_n + \alpha_n^2 + \beta_n^2\right), \tag{3}$$

$$Q_{\mathrm{sca}} = \frac{2}{ka}\sum_{n=0}^{\infty}\varepsilon_n\left(\alpha_n^2 + \beta_n^2\right), \tag{4}$$

where $\varepsilon_n = 2 - \delta_{n0}$, and δ_{ij} is the Kronecker delta function.

The scattering coefficients of the cylinder C_n are expressed in terms of cylindrical Bessel, Neumann and Hankel functions and their derivatives, and they are available in standard literature [51].

Consider now the case of a VE-PMMA cylindrical material (having a density $\rho_{\mathrm{PMMA}} = 1191\ \mathrm{kg/m^3}$, with a speed of sound for the compressional $c_{L,\mathrm{PMMA}} = 2690$ m/s and shear waves $c_{S,\mathrm{PMMA}} = 1340$ m/s), immersed in (non-viscous) water. Absorption by the cylinder is modeled by introducing complex wave numbers with normalized absorption coefficients independent of frequency, which holds for linear viscoelasticity [17,54]. This behavior is typically encountered for various polymeric materials [13]. The normalized absorption coefficient for the longitudinal wave is given as [21] $\gamma_{L,\mathrm{PMMA}} = 0.0119$, and the one for the shear wave is varied in the range $0 \leqslant \gamma_{S,\mathrm{PMMA}} \leqslant 1$. Eqs. (1)–(4) are evaluated numerically by developing a MATLAB program used in the limit $0 < ka \leqslant 1$, where the anomalous effects have been observed. Adequate truncation of the series in Eqs. (1)–(4) has been applied such that n_{max} largely exceeds ka so that the numerical convergence of the series is warranted.