Ultrasonics 74 (2017) 62–71

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Axial acoustic radiation force on rigid oblate and prolate spheroids in Bessel vortex beams of progressive, standing and quasi-standing waves

F.G. Mitri

Chevron, Area 52 Technology – ETC, Santa Fe, NM 87508, United States

ARTICLE INFO

Article history:
Received 27 July 2016
Received in revised form 26 September 2016
Accepted 29 September 2016
Available online 30 September 2016

Keywords:
Acoustic radiation force
Spheroid
Bessel beam
Tractor beam
Acoustical tweezers
Partial-wave series expansion (PWSE)

ABSTRACT

The analysis using the partial-wave series expansion (PWSE) method in spherical coordinates is extended to evaluate the acoustic radiation force experienced by rigid oblate and prolate spheroids centered on the axis of wave propagation of high-order Bessel vortex beams composed of progressive, standing and quasi-standing waves, respectively. A coupled system of linear equations is derived after applying the Neumann boundary condition for an immovable surface in a non-viscous fluid, and solved numerically by matrix inversion after performing a single numerical integration procedure. The system of linear equations depends on the partial-wave index n and the order of the Bessel vortex beam m using truncated but converging PWSEs in the least-squares sense. Numerical results for the radiation force function, which is the radiation force per unit energy density and unit cross-sectional surface, are computed with particular emphasis on the amplitude ratio describing the transition from the progressive to the pure standing waves cases, the aspect ratio (i.e., the ratio of the major axis over the minor axis of the spheroid), the half-cone angle and order of the Bessel vortex beam, as well as the dimensionless size parameter. A generalized expression for the radiation force function is derived for cases encompassing the progressive, standing and quasi-standing waves of Bessel vortex beams. This expression can be reduced to other types of beams/waves such as the zeroth-order Bessel non-vortex beam or the infinite plane wave case by appropriate selection of the beam parameters. The results for progressive waves reveal a tractor beam behavior, characterized by the emergence of an attractive pulling force acting in opposite direction of wave propagation. Moreover, the transition to the quasi-standing and pure standing wave cases shows the acoustical tweezers behavior in dual-beam Bessel vortex beams. Applications in acoustic levitation, particle manipulation and acousto-fluidics would benefit from the results of the present investigation.

1. Introduction

Radiation force simulations in acoustical tweezers applications have become an indispensable tool to scientific research dealing with the interaction of acoustical waves (or beams) with objects, especially in acoustofluidics [1,2], particle manipulation [3] and acoustical tweezers [4–18], elasticity imaging [19], and acoustic levitation [20–25] to name a few applications. Numerical computations are essential as they provide guidance for optimal experimental design purposes. Furthermore, since experiments require adequate instrumentation and hardware equipment (which may be often expensive), and are time-consuming so in practice only a limited number can be performed on well-defined geometries, most investigations resort initially to numerical computations of the radiation force of acoustical waves exerted on an object placed along their path. By developing fast and accurate computational tools, precise radiation force modeling of any scenario of interest can be made possible. This includes extreme cases satisfying certain limits which may not be entirely attainable experimentally; for example, the completely rigid or soft particle cases. Clearly, there is a continuing need for improved modeling, analytical theories and computer simulations, especially when more complex (non-circular) geometries and/or beam profiles of arbitrary wavefronts are considered.

Various investigations limited to the long wavelength limit examined the acoustic radiation force on disks [26–29] and prolate spheroids [30] in plane progressive and standing waves. Moreover, the finite-element method [31] (FEM) has been utilized by means of the shape perturbation method to evaluate the acoustic radiation force on a (non-spherical) rigid spheroid in *plane* standing waves.

Notice, however, that the shape perturbation method is only applicable to a near-spherical particle, and leads to significant errors for the cases of moderately to highly elongated spheroids.

E-mail address: F.G.Mitri@ieee.org

http://dx.doi.org/10.1016/j.ultras.2016.09.022
0041-624X/

An analysis based on the boundary element method (BEM) [32] has been also developed, in which the acoustic radiation force experienced by non-spherical particles has been computed for cases where the target's dimensions are much smaller than the wavelength of the incident illuminating plane waves (i.e. Rayleigh limit). The lack of adequate data beyond the long-wavelength limit as well as the restriction to the infinite plane wave case provided the impetus to further develop an improved solution [33] based on the modal-matching method in spherical coordinates to compute the acoustic radiation force on rigid oblate and prolate spheroids applicable to zeroth-order Bessel beams, including the infinite plane wave case.

In contrast to plane (or Gaussian) waves, Bessel beams possess several features, such as the ability to reform after encountering an obstruction [34,35]. They also have a limited-diffraction capability such that they maintain a relatively long depth-of-field as they propagate in space [36]. Due to these advantages, such beams are well established both theoretically and experimentally, and related innovative applications in fundamental and applied physics are increasingly expanding in various fields.

The zeroth-order Bessel beam is of non-vortex type, while the higher-order is of vortex (i.e. spiral or helicoidal) type, such that its incident velocity potential (or pressure) field varies according to $\exp(im\phi)$, where m is any (real) integer number known as the order or the topological charge, and ϕ is the azimuthal angle. This effect can be clearly emphasized by considering a computational example for the intensity vector field $\mathbf{I} = \Re\{p^*\mathbf{v}\}$ of a Bessel vortex beam, where the parameters p and $\mathbf{v}$ denote the incident linear pressure and vector velocity, respectively. The superscript $*$ denotes the conjugate of a complex number. The results are displayed in Fig. 1 for the computational plots (with stereographic projections in the bottom plane superimposed on the cross-sectional profile of the beam) corresponding to the spatial distribution of the intensity vector field (shown by the arrows) of a Bessel beam sampled uniformly on the surface of the spheroid. The plots in panels (a)–(c) correspond to a zeroth-order Bessel (non-vortex) beam where a maximum intensity at the center of the beam is produced, whereas panels (d)–(f) are for a first-order Bessel spiral (or vortex) beam with a unit topological charge (i.e., $m = 1$) where a null [37] (or phase singularity along the axis of wave propagation) is manifested at the center of the beam. The incident Bessel waves illuminate spheroidal particles with different aspect ratios centered on the axis of the beam. The particle in panels (a), (d) has the shape of an oblate spheroid, whereas in panels (b), (e), the particle is a sphere. In panels (c), (f), the particle has the shape of a prolate spheroid. The characteristic of the vortex is clearly shown in panels (d)–(f) with an intensity null at the center of the beam. Depending on the vortex helicity ($m = \pm 1$), the arrows representing the intensity vector field can be directed counter-clockwise or clockwise, respectively.

While there exists significant literature on the interaction of acoustical Bessel beams with spherical objects including several investigations focused on the (arbitrary) scattering [38–40] and radiation forces [41–45], the spheroidal object has been only recently considered from the standpoint of acoustic scattering [46–49] (using also the T-matrix formalism [46,50]) and radiation force theories [33,51]. As the properties of the higher-order beam solutions (using progressive waves) [51] differ considerably from the fundamental (zeroth-order) case [33], it is of some importance to develop an appropriate analytical formalism to analyze and compute numerically the acoustic radiation force of Bessel vortex beams of standing and quasi-standing waves exerted on spheroids. Typically, standing and quasi-standing waves are obtained by counter-propagating two (or more) Bessel vortex beams. This configuration is also known as the dual-beam tweezers as suggested in the original version of acoustical tweezers [52]. Note that the formalism used for Bessel vortex beams of progressive waves [51] is not applicable to the dual-beam configuration, nor can it be considered as an approximate solution because the localizing force strengths for the representative standing and quasi-standing wave modes depend on the spatial phase which is not the case for progressive waves. Moreover, the standing and quasi-standing waves in the dual-beam configuration form stable positioning in acoustical potentials, a situation that can be hardly achieved using the single-beam of progressive waves. Such limitations provide the motivation and impetus to undertake the present analysis, and develop an analytical method that encompasses all types of waves in Bessel vortex beams, ranging from progressive, standing to quasi-standing waves.

In this analysis, the acoustic scattering of Bessel vortex beams of quasi-standing waves by a rigid (sound impenetrable) oblate or prolate spheroid (Fig. 2) is first solved first using the partial-wave series expansion (PWSE) method in spherical coordinates. Then, it is used to derive a generalized analytical expression for the axial acoustic radiation force (i.e., acting along the direction of wave propagation) that is applicable to the cases of progressive, quasi-standing and standing waves in Bessel vortex beams. It is important to note here that such spheroidal (convex-like) surfaces present a serious challenge because the method of separation of variables (used to evaluate the scattering and subsequently the radiation force) becomes inapplicable. In other words, the spherical-wave functions used in the method of separation of variables become non-orthogonal on the object's surface, consequently, adequate convergence and accuracy of the results can be hardly achieved. Nonetheless, this difficulty is resolved by using the PWSE using an improved methodology based on modal matching. The method requires solving a system of linear equations by matrix inversion procedures [i.e. Eq. (15) in the following] which depends on the partial-wave index n and the order m of the Bessel vortex beam. For the case considered in the present manuscript, the Neumann boundary condition for a rigid immovable surface is satisfied in the least-squares sense with negligible truncation numerical error. This original semi-analytical approach developed for Bessel vortex beams is demonstrated for finite oblate and prolate spheroids, where the mathematical functions describing the spheroidal geometry are written in a form involving single angular (polar) integrals that are numerically computed using Gauss-Legendre quadratures. Then, the axial radiation force is evaluated stemming from an analysis of the far-field scattering, with particular emphasis on the aspect ratio (i.e., the ratio of the major axis over the minor axis of the spheroid), the half-cone angle β and the order m of the Bessel vortex beam, as well as the dimensionless size parameter kb. Moreover, the radiation force function expression reduces to progressive or equi-amplitude standing waves depending on the choice of the coefficient R, defined as an amplitude-ratio factor of the waves (Section 2).

2. Theoretical formalism

Consider an acoustical monochromatic high-order Bessel vortex beam of order m propagating in a nonviscous fluid, and incident upon a spheroid centered on its axis of wave propagation [i.e., end-on incidence (Fig. 2)]. The spheroid has an equatorial radius b (known as the semi-minor axis), and a is the distance from the center to the pole along the symmetry axis z, corresponding to the semi-major axis.

The incident field is assumed to be composed of two counter-propagating Bessel vortex beams of the same order but with different amplitudes. This generally defines an acoustic velocity potential field of quasi-standing waves.

In a system of spherical coordinates (r, θ, ϕ) with its origin chosen at the center of the spheroid, the incident velocity potential