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Vibration characteristics of an ultrasonic transducer of two piezoelectric discs

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ABSTRACT

This paper considers the influence of the different thickness of the piezoelectric discs used in assembly of an ultrasonic sandwich transducer. The transducer consists of two piezoelectric discs with different thickness between 0 and 2.0 mm and with same diameter 28 mm. Its vibration characteristics of the radial and axial motions were investigated theoretically and experimentally in axisymmetric vibration modes. Theoretically, the differential equations of piezoelectric motions were solved to produce characteristic equations that provided natural frequencies and mode shapes. The range of the fundamental frequency of radial in-plane mode is 80–360 kHz and that of the axial out-of-plane mode is 600–1200 kHz. Experimentally, the natural frequencies and the radial in-plane motion were measured using an impedance analyzer and an in-plane laser interferometer, respectively. The results of the theoretical analysis were compared with those of a finite-element analysis and experiments, and the theoretical analysis was verified on the basis of this comparison. It was concluded that the natural frequencies of the radial modes of the transducer were not affected by the stack and thickness of piezoelectric discs; however, those of the thickness modes were affected by the stack and thickness of the piezoelectric discs.

1. Introduction

The purpose of this work is to explore the effect of the stack of piezoelectric discs on the vibration characteristics of an ultrasonic transducer. The discs have different thicknesses and the transducers could operate in the frequency range about one to several hundred kHz.

Piezoelectricity-based transducers are widely used, especially in ultrasonic sensors [1] and actuators [2]. They adopt electromechanical transduction mechanisms, which convert mechanical vibrations into electric signals or vice versa [3]. Sandwich transducers, such as these considered here are often used in ultrasonic cleaners [4]. These transducers are designed as resonance devices [5] to optimize the displacement amplitude of the vibrating displacement amplitude of the vibrating surface and hence the effectiveness of operation.

A piezoelectric disc is usually a main element of piezoelectric transducers, the vibration characteristics of which are well known [6]. Piezoelectric discs polarized in the thickness direction vibrate not only in the thickness direction but also in the radial direction owing to Poisson's ratio [7]. In some cases, piezoelectric transducers consist of coupled piezoelectric and elastic discs or multiple piezoelectric discs [8]. Elastic or piezoelectric discs are coupled with a piezoelectric disc of a transducer in order to decrease the fundamental frequency of the transducer [9–12]. A bolt-clamped Langevin-type transducer is a typical example of transducers containing piezoelectric discs with added elastic blocks [13].

Guo et al. predicted the vibration characteristics of piezoelectric discs by finite element methods and modal analysis techniques [14], where five types of modes, including radial mode, were identified according to their mode shape characteristics. Heylinger and Ramirez considered free vibration of laminated circular piezoelectric discs in order to compute natural frequencies by introducing a numerical model [15], which combined approximations of one-dimensional finite elements in the thickness direction and analytic functions in the plane. Wang et al. investigated the radial vibration of piezoelectric hollow-disc stack to obtain resonance frequencies [16]. Laoratanakul and Uchino fabricated and studied the laminated piezoelectric for the purpose of designing a high power transformer [17]. In these stacked transducers, the piezoelectric discs or plates are of the same sizes in the thickness and planar dimensions.

Different types of piezoelectric stack transducers can be devised to generate multiple resonances around the fundamental mode. One type is a stack of two or more piezoelectric discs with same

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thickness but different radius. Another is a stack of those with same radius but different thickness. Before studying the stacked piezoelectric discs with different radius, this paper considers a stack of two piezoelectric discs with same radius but different thickness. Unlike the piezoelectric transducer laminated with an elastic disc [9] or a piezoelectric disc [10], this paper considers the effects of electrical properties of the laminated piezoelectric disc of various thicknesses to the ultrasonic transducer. The transducers of piezoelectric disc stack are used in ultrasonic cleaners and SONARs.

Section 2 describes theoretical analysis. Theoretically, the differential equations of piezoelectric motions are derived in terms of radial and axial displacements and electric potential. Governing equations satisfying boundary conditions are solved to obtain characteristic equations. Section 3 presents numerical results. Based on the theoretical analysis, the vibration characteristics of the ultrasonic transducer are calculated numerically to provide natural frequencies and mode shapes. The results are complemented by a finite-element analysis. Section 4 deals with experiments. The natural frequencies are measured using an impedance analyzer. Radial in-plane motions are measured using an in-plane laser interferometer. In Section 5 the results of the theoretical analysis are compared with those of a finite-element analysis and experiments; moreover, the theoretical analysis is verified on the basis of this comparison.

2. Theoretical analysis

An ultrasonic transducer composed of two piezoelectric discs is schematically shown in Fig. 1; a is the radius, and l_1 and l_2 are the thicknesses of the piezoelectric discs. The thickness of the adhesive between the discs is about 10 μm and it is assumed to be negligible in comparison with PZT disc thickness larger than 0.8 mm. The transducer has uniform electrodes on each surface at $z = 0$, l_1, and l_2. The differential equations of piezoelectric motions are derived in terms of radial and axial displacements and electric potential, and they are solved to obtain characteristic equations by satisfying the boundary conditions. Radial in-plane modes and axial out-of-plane modes are considered in the analysis, and the latter is needed to higher frequency resonances. The analysis result is used to predict the trend of the vibration characteristics depending on the geometric parameters.

2.1. Radial in-plane modes

The piezoelectric constitutive equations are summarized in the Appendix A. The equation of motion derived from force equilibrium in the radial direction is as follows [18]:

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial^2 u}{\partial t^2} \tag{1}$$

where ρ is the mass density. Inserting Eqs. (A8a) and (A8b) in Eq. (1) yields the following governing equation for the piezoelectric discs 1 and 2:

$$\frac{\partial^2 u_i}{\partial r^2} + \frac{1}{r}\frac{\partial u_i}{\partial r} - \frac{u_i}{r^2} = \frac{1}{C_R^2}\frac{\partial^2 u_i}{\partial t^2} \quad (i = 1, 2) \tag{2}$$

where $C_R = \left[c_{11}^p/\rho\right]^{1/2}$.$C_R$ in the wave equation (Eq. (2)) is the wave propagation speed in the radial direction.

The radial displacement at the center is zero and circumferential faces are free of traction; therefore,

$$u_i(0, z, t) = 0 \quad \text{at } r = 0 \quad (i = 1, 2) \tag{3a}$$

$$\sigma_{ri}(a, z, t) = 0 \quad \text{at } r = a \quad (i = 1, 2) \tag{3b}$$

When the voltage applied to the electrodes is a harmonic function of time t with frequency ω, the displacement u and electric potential ϕ are regarded as harmonic functions of time with the same frequency. It was reported earlier that the radial displacement u is not dependent on the axial coordinate z in a piezoelectric transducer driven by a uniform electric field in the thickness direction [6]. Therefore, $u(r, t)$ and $\sigma_r(r, t)$ can be expressed via the separation of variables as follows:

$$u_i(r, t) = U_i(r)e^{j\omega t} \tag{4a}$$

$$\sigma_{ri}(r, t) = \overline{\sigma}_{ri}(r)e^{j\omega t} \quad (i = 1, 2) \tag{4b}$$

The electric field is considered to vary linearly in the thickness direction.

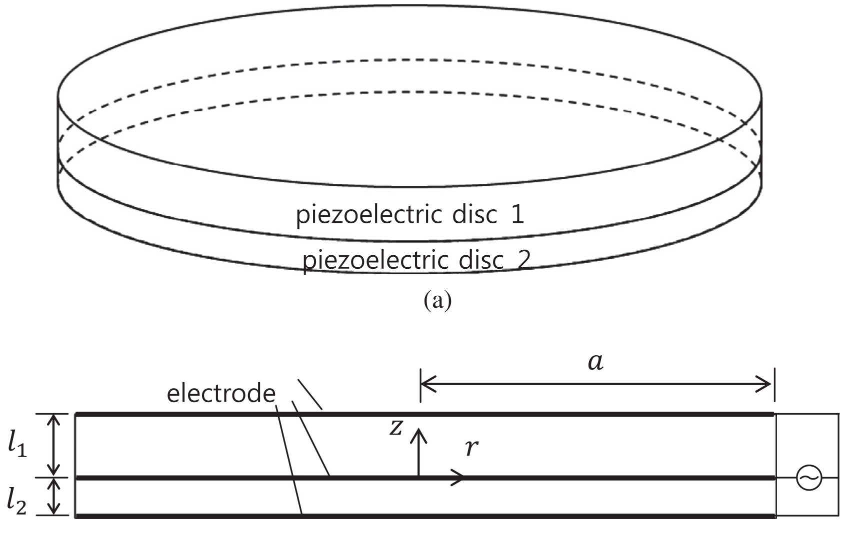

Fig. 1. Schematic diagram of an ultrasonic transducer of two piezoelectric discs; (a) two discs, (b) coordinates.