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A reverberation-ray matrix method for guided wave-based non-destructive evaluation

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ABSTRACT

The paper presents an application of the reverberation-ray matrix (RRM) method for guided wave-based non-destructive evaluation (NDE). An exact analytical model for elastic wave propagation in multilayered anisotropic composites is developed with the RRM method. Dispersion curves, namely phase and group velocities varying with frequencies, can be calculated based on the analytical model, which are critical to the guided wave-based NDE. In addition, the characteristics of the guided wave propagation along different directions in laminated composites with different anisotropic degrees are investigated. Finally, the results obtained from the model are verified by finite element simulations.

1. Introduction

Nowadays the amount of composite material used in aircraft structures is gradually increasing, from secondary to primary structures, up to 50% in the Boeing's B787 and 53% in the Airbus' A350 XWB, because of their excellent stiffness and strength to weight ratios [1]. Delamination is one of the most common and dangerous damages in laminated composites, which can significantly reduce structural stiffness and cannot observe from outside. The damage propagates accompanied with other damage types under fatigue loads, and leads to structural failure at last [2]. Guided waves those can propagate long distances and are sensitive to delamination damage are appropriate choices to enhance laminate's integrity [3]. However, due to geometry boundaries in laminates, multiple dispersed wave modes exist, which makes the guided wave-based NDE complex [4]. In addition, the wave characteristics along different directions are different because of the anisotropy of laminates. Therefore the damage inspection in laminates is more difficult than that in isotropic metallic materials [5].

Dispersion curves, namely phase and group velocities varying with frequencies, provide important information on wave propagation in waveguides, which are critical to the guided wave-based NDE. In the analytical methods, the most commonly used are the transfer matrix method and global matrix method [6–9]. The transfer matrix is a simple and direct method to implement that the matrix order is unchanged regardless of the number of layers in laminates. However, when evanescent waves exist in the cases of high frequencies or long propagation distances, the solutions become unstable. The global matrix method overcomes the shortcoming of the transfer matrix method by modifying the origins of the bulk waves in each layer according to their propagation directions. In addition to the accurate methods, truncated Legendre orthogonal polynomials can also be used to approximate the displacement fields of the cross section and then calculate the dispersion curves [10]. In the numerical methods, the finite element method (FEM) is a well-developed one which can solve various complex problems in practice, including wave propagation. The semi-analytical FEM is used to investigate the guided wave propagation in waveguides with finite cross sections and multilayered structures, in which wave motions in the propagation direction are formulated as plane waves, and the cross sections are discretized into finite elements [11–13]. In the full FEM, a unit cell representing a waveguide is discretized into finite elements, and then dispersion curves can be calculated by introducing Bloch boundary conditions on the unit cell [14–16]. The FEM and spectral element method (SEM) can also be used to simulate the process of wave propagation and extract the characteristics [17,18]. However, the numerical methods above require elements to approximate the displacement fields, which causes the increase of computation effort at higher frequencies.

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The RRM is an alternative analytical method to obtain dispersion curves in multilayered anisotropic composites, which was proposed by Howard and Pao in 1998 [19]. Due to the settings of dual coordinate systems, elements in the phase matrix can always be kept as exponentially decaying functions when evanescent waves exist. Therefore the method avoids the numerical instability problem in the cases of high frequencies or long propagation distances, which is occurred in the transfer matrix method [20–22]. The RRM method is appropriate to calculate transient responses because the inverse Fourier transform with singularities can be avoided by the Neumann series expansion [19–21]. The method is also suitable to investigate wave propagation characteristics due to its unified formulation and numerical stability [22,23]. However, few studies have been focused on this direction, especially the application for guided wave-based NDE.

The paper presents an application of the RRM method for guided wave-based NDE. An exact three dimensional analytical model for elastic wave propagation in laminated composites is developed with the RRM method. The expression of group velocity is derived analytically. Wave propagation characteristics are calculated along different directions, which are different from those in isotropic metallic plates. The remainder of this paper comprises four major parts. The basic principle of the RRM method is introduced and the problem is formulated in Section 2. The finite element models are developed to simulate wave propagation and verify the RRM method in Section 3. In Section 4, the calculation results are compared and analyzed. Finally, this paper is concluded in Section 5.

2. RRM method for wave propagation

The RRM method is used to model guided wave propagation in laminated composites. Displacements and out-of-plane stresses are chosen as state variables to express state equations, which makes the problem easier to solve.

2.1. Governing equations

The laminated composite is composed of unidirectional layers which located in the xy-plane along different directions and stacked along the z-axis as shown in Fig. 1. Due to the up and down surfaces and the anisotropy of each layer, the guided waves in laminates present three characteristics those are dispersion, existence of multiple modes and velocity anisotropy.

In order to describe the wave motions at any frequency accurately, three dimensional equations on the assumptions of small deformation and linear elasticity are used to model laminated composite. In each layer, the geometry, constitutive and equilibrium equations can be respectively written as

$$\boldsymbol{\varepsilon} = \mathbf{L}^{\mathrm{T}}\mathbf{u}, \quad \boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}, \quad \mathbf{L}\boldsymbol{\sigma} = \rho\ddot{\mathbf{u}} \tag{1}$$

where ρ is the material density, and

$$\mathbf{u} = [u_x, u_y, u_z]^{\mathrm{T}}, \quad \boldsymbol{\varepsilon} = [\varepsilon_x, \varepsilon_y, \varepsilon_z, \varepsilon_{yz}, \varepsilon_{xz}, \varepsilon_{xy}]^{\mathrm{T}},$$
$$\boldsymbol{\sigma} = [\sigma_x, \sigma_y, \sigma_z, \sigma_{yz}, \sigma_{xz}, \sigma_{xy}]^{\mathrm{T}} \tag{2}$$

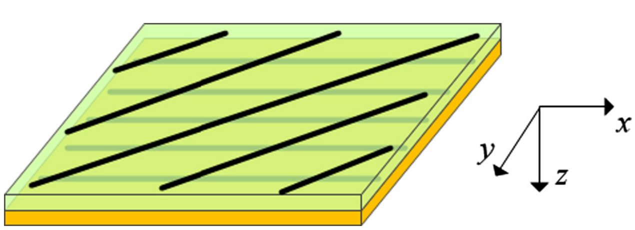

Fig. 1. Laminated composite in a global coordinate system.

represent the displacement, strain and stress vectors respectively, and

$$\mathbf{D} = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ D_{21} & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ D_{31} & D_{32} & D_{33} & D_{34} & D_{35} & D_{36} \\ D_{41} & D_{42} & D_{43} & D_{44} & D_{45} & D_{46} \\ D_{51} & D_{52} & D_{53} & D_{54} & D_{55} & D_{56} \\ D_{61} & D_{62} & D_{63} & D_{64} & D_{65} & D_{66} \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \tag{3}$$

represent the stiffness matrix and differential operator matrix. The generalized Hooke's law is used to describe the material anisotropy. For harmonic plane guided waves along the angle α with respect to the x-axis in laminates, the field variables can be written as

$$\mathbf{u} = \hat{\mathbf{u}}(z)\mathrm{e}^{i\omega\left(\frac{x}{c_{\mathrm{p}}}\cos\alpha + \frac{y}{c_{\mathrm{p}}}\sin\alpha - t\right)}, \quad \boldsymbol{\varepsilon} = \hat{\boldsymbol{\varepsilon}}(z)\mathrm{e}^{i\omega\left(\frac{x}{c_{\mathrm{p}}}\cos\alpha + \frac{y}{c_{\mathrm{p}}}\sin\alpha - t\right)},$$
$$\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}(z)\mathrm{e}^{i\omega\left(\frac{x}{c_{\mathrm{p}}}\cos\alpha + \frac{y}{c_{\mathrm{p}}}\sin\alpha - t\right)} \tag{4}$$

where ω is the angular frequency, and c_{p} is the phase velocity. The commonly used wave number k is replaced by c_{p} because velocities are more concerned than wave numbers in guided wave-based NDE. Combining Eqs. (1) and (4), the governing equation of wave motions can be expressed as

$$\frac{\mathrm{d}\hat{\mathbf{v}}}{\mathrm{d}z} = \mathbf{A}(c_{\mathrm{p}}, \alpha, \omega)\hat{\mathbf{v}} \tag{5}$$

where

$$\hat{\mathbf{v}} = [\hat{u}_x, \hat{u}_y, \hat{u}_z, \hat{\sigma}_{xz}, \hat{\sigma}_{yz}, \hat{\sigma}_{zz}]^{\mathrm{T}} \tag{6}$$

$$\mathbf{A} = \begin{bmatrix} -\mathrm{i}\omega\tilde{\mathbf{D}}_{33}^{-1}\mathbf{M}_3 & \tilde{\mathbf{D}}_{33}^{-1} \\ \omega^2\left(-\rho\mathbf{I}_3 + \frac{\cos\alpha}{c_{\mathrm{p}}}\mathbf{M}_1 + \frac{\sin\alpha}{c_{\mathrm{p}}}\mathbf{M}_2 - \mathbf{M}_3^{\mathrm{T}}\tilde{\mathbf{D}}_{33}^{-1}\mathbf{M}_3\right) & -\mathrm{i}\omega\mathbf{M}_3^{\mathrm{T}}\tilde{\mathbf{D}}_{33}^{-1} \end{bmatrix} \tag{7}$$

$$\mathbf{M}_i = \frac{\cos\alpha}{c_{\mathrm{p}}}\tilde{\mathbf{D}}_{i1} + \frac{\sin\alpha}{c_{\mathrm{p}}}\tilde{\mathbf{D}}_{i2}, \quad \tilde{\mathbf{D}}_{ij} = \begin{bmatrix} c_{1i1j} & c_{1i2j} & c_{1i3j} \\ c_{2i1j} & c_{2i2j} & c_{2i3j} \\ c_{3i1j} & c_{3i2j} & c_{3i3j} \end{bmatrix} \tag{8}$$

The state vector $\hat{\mathbf{v}}$ represents the displacements and out-of-plane stresses. The constant c_{klmn} is an alternative way to express the material stiffness D_{ij} in Eq. (3) in the four-order form. $\mathbf{I}_3$ is the 3-by-3 unit matrix. The solution of Eq. (5) is

$$\hat{\mathbf{v}}(z) = \boldsymbol{\Phi}\mathrm{e}^{\boldsymbol{\Lambda}z}\mathbf{w} = [\boldsymbol{\Phi}_- \quad \boldsymbol{\Phi}_+]\begin{bmatrix} \mathrm{e}^{\boldsymbol{\Lambda}_- z} & 0 \\ 0 & \mathrm{e}^{\boldsymbol{\Lambda}_+ z} \end{bmatrix}\begin{bmatrix} \mathbf{a} \\ \mathbf{d} \end{bmatrix} \tag{9}$$

where

$$\boldsymbol{\Phi} = [\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_6], \quad \boldsymbol{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_6) \tag{10}$$

λ and $\boldsymbol{\varphi}$ represent the eigenvalue and eigenvector of the matrix $\mathbf{A}$ respectively. The vector $\mathbf{w}$ contains the undetermined coefficients. The eigenvalues exist in pairs, which have opposite signs, because waves in material can propagate in two opposite directions. Here the two waves in pairs are assigned to the groups with the signs '+' and '–' respectively, corresponding to the undetermined coefficient vectors $\mathbf{a}$ and $\mathbf{d}$, which is the critical step to keep the computation numerically stable and illustrated in detail in Section 2.3.

2.2. Dual coordinate systems

The setting of dual coordinate systems is the core difference between the RRM method and others as shown in Fig. 2, where the x- and z-axes are both in the opposite directions and the y-axes are in the same. h^j is the thickness of the layer j with the bound-