Ultrasonics 77 (2017) 95–99

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Unusual energy properties of leaky backward Lamb waves in a submerged plate

I.A. Nedospasov [a], V.G. Mozhaev [b,*], I.E. Kuznetsova [a]

[a] Kotelnikov Institute of Radio Engineering and Electronics, Russian Academy of Sciences, Mokhovaya 11-7, Moscow 125009, Russia
[b] Physics Faculty, Lomonosov Moscow State University, Leninskie Gory 1-2, Moscow 119991, Russia

ARTICLE INFO

Article history:
Received 7 November 2016
Received in revised form 25 January 2017
Accepted 30 January 2017
Available online 1 February 2017

Keywords:
Leaky backward waves
Lamb modes
Submerged plate
Energy flux
Energy velocity
Group velocity
Existence region
Rayleigh's principle

ABSTRACT

It is found that leaky backward Lamb waves, i.e. waves with negative energy-flux velocity, propagating in a plate submerged in a liquid possess extraordinary energy properties distinguishing them from any other type of waves in isotropic media. Namely, the total time-averaged energy flux along the waveguide axis is equal to zero for these waves due to opposite directions of the longitudinal energy fluxes in the adjacent media. This property gives rise to the fundamental question of how to define and calculate correctly the energy velocity in such an unusual case. The procedure of calculation based on incomplete integration of the energy flux density over the plate thickness alone is applied. The derivative of the angular frequency with respect to the wave vector, usually referred to as the group velocity, happens to be close to the energy velocity defined by this mean in that part of the frequency range where the backward mode exists in the free plate. The existence region of the backward mode is formally increased for the submerged plate in comparison to the free plate as a result of the liquid-induced hybridization of propagating and nonpropagating (evanescent) Lamb modes. It is shown that the Rayleigh's principle (i.e. equipartition of total time-averaged kinetic and potential energies for time-harmonic acoustic fields) is violated due to the leakage of Lamb waves, in spite of considering nondissipative media.

1. Introduction

The wave modes with opposite directions of the phase velocity and the energy flux, alternatively referred to as waves with negative group velocity or negative energy-flux velocity and also shortly as backward waves, attract a great deal of attention in the various fields of wave physics. It is so, particularly, in acoustics, where plate modes of Lamb type and of pure-shear polarization can have the negative group velocity in limited ranges of frequencies and thicknesses in isotropic and anisotropic plates [1–9]. In spite of a few efforts of analysis of leaky backward Lamb waves [10–16], no studies until now have directly examined their energy properties, although these properties belong to the most fundamental ones for waves of any nature. The present paper is focused on exploring such properties. It is found that the total time-averaged energy flux (i.e. energy flux density integrated over the depth) of waves, treated here, is equal to zero while the mutually opposite longitudinal components of local energy fluxes in a plate and in a liquid are nonzero. This property gives rise to the fundamental question of how to define correctly the energy velocity in such an unusual case. Another open question is whether it is possible to calculate this velocity by differentiating the dispersion curves in the same way as the group velocity is found. The third important question is about the validity of the Rayleigh's principle of equipartition of the total time-averaged kinetic and potential energies in the system under study. The answers to these questions are given in the present paper. In the beginning, we overview the main features of leaky backward Lamb waves that are necessary to study their energy properties. The first symmetric Lamb mode (S_1) in a free isotropic plate, as is known, can be backward wave in a limited range of frequencies [3]. It is just this mode that is considered below in the case of plate being in contact with the same liquid on both sides.

2. Decay constants and phase velocities for backward leaky waves

The equation of motion in a solid for harmonic waves varying with time as $\exp(-i\omega t)$ has the form

$$-i\omega\rho v_i = \partial_j T_{ij}, \quad (1)$$

* Corresponding author.
E-mail addresses: ianedospasov@mail.ru (I.A. Nedospasov), vgmozhaev@mail.ru (V.G. Mozhaev), kuziren@yandex.ru (I.E. Kuznetsova).

http://dx.doi.org/10.1016/j.ultras.2017.01.025
0041-624X/

where ω is the angular frequency, ρ is the mass density, v_i are the particle velocity components, $\partial_j = \partial/\partial x_j$, x_j are Cartesian coordinates. Here and hereafter summation over repeated subscripts $i, j, k, l = 1-3$ is implied. The stress tensor T_{ij} is related to the strain tensor S_{kl} by the Hook's law, $T_{ij} = c_{ijkl}S_{kl}$, where c_{ijkl} is the elastic stiffness tensor, $S_{kl} = (\partial_l u_k + \partial_k u_l)/2$, u_k are the particle displacement components. The time-harmonic waves in a non-viscous liquid are described by the Helmholtz equation $\Delta\phi + k^2\phi = 0$, where Δ is the Laplacian, ϕ is the potential of particle velocity, $v_i = \partial_i\phi$, k is the wavenumber. The coordinate perpendicular to the plate surface is denoted as z axis, the coordinate along the plate as x axis. The 2-dimensional wave-propagation problem depending solely on these two coordinates is further considered. The boundary conditions at the interface between the plate and the liquid are the equality $T_{xz} = 0$ and the continuity of the normal component of displacement u_z and the traction force $T_{zz} = -p$ [17], where p is the acoustic pressure, $p = -\rho_{\mathrm{lq}}\partial\phi/\partial t$, ρ_{lq} is the liquid mass density.

Lamb waves propagating in the submerged plate faster than bulk waves in the liquid radiate their energy into outer spaces and for this reason they become leaky waves and so are decaying along the plate. The decay constant α can be found by the perturbation method [17], assuming that the liquid-loading effect is small

$$\alpha = [Z_0(d) - Z_0(-d)]\omega^2|u_z(d)|^2/(4\langle\bar{P}_x^{\mathrm{R}}\rangle), \tag{2}$$

where $\langle\bar{P}_x^{\mathrm{R}}\rangle$ is the time-averaged energy flux density integrated across the plate, $\langle\bar{P}_x^{\mathrm{R}}\rangle = \int_{-d}^{d}\bar{P}_x^{\mathrm{R}}dz$, $\bar{P}_i^{\mathrm{R}} = \mathrm{Re}(\bar{P}_i)$, $\bar{P}_i = -T_{ij}v_j^*/2$, d is the half thickness of plate, $u_z(d)$ is the normal-displacement amplitude at the surface, $Z_0(\pm d) = \pm\rho_{\mathrm{lq}}V_{\mathrm{c}}V_{\mathrm{L}}/\sqrt{V_{\mathrm{L}}^2 - V_{\mathrm{c}}^2}$ is the surface mechanical impedance of liquid, V_{c} is the phase velocity of compressional wave in the liquid, V_{L} is the Lamb wave phase velocity. As evident from Eq. (2), the quantities α and $\langle\bar{P}_x^{\mathrm{R}}\rangle$ should be of the same sign. Since $\langle\bar{P}_x^{\mathrm{R}}\rangle$ is negative with respect to the wave vector for the backward waves, the coefficient α is also negative for these waves. Note that the right-hand side of Eq. (2) is determined by the unperturbed solution of the wave propagation problem for the free plate. However all other formulas given below include the wave characteristics for submerged plates.

The change in sign of α for the backward leaky waves has a critical effect on the structure of radiation in the liquid. The spatial dependence of this radiation has the form $\exp(ik_x x + ik_z z)$, where $k_x = k_{\mathrm{L}} + i\alpha$, $k_{\mathrm{L}} = \omega/V_{\mathrm{L}}$. The projection k_z of the complex wave vector, $k_z = \mathrm{Re}(k_z) + i\beta$, is found from the Helmholtz equation

$$\mathrm{Re}(k_z) = \pm\sqrt{k^2 - k_{\mathrm{L}}^2 + \alpha^2 + \beta^2}, \quad \beta = -\alpha k_{\mathrm{L}}/\mathrm{Re}(k_z). \tag{3}$$

Formulas (3) show that the constant β has the same sign as $\mathrm{Re}(k_z)$, that is, the wave radiation in the liquid falls down along the outer normal to the plate surface.

Thus, in contrast to the forward leaky waves, the backward ones decay along the plate in the opposite direction to the wave vector and their radiation in the liquid decreases rather than increases with distance from the plate. These conclusions are in agreement with those obtained analytically in Ref. [13] although here they are based on other arguments. The negative sign of α is also in agreement with numerical calculations in Ref. [11]. One should note that there are partially analogous theoretical and experimental studies [18–20], pertaining to backward quasisymmetric waves (denoted as S_{2b}) on water-loaded empty shells. These studies reach the same conclusion as above; that $\alpha < 0$.

The dependencies of phase velocities of the S_1 mode on the product of frequency f and thickness h, calculated by us for aluminum plate which is free (curves *1* and *2*) or submerged in water (curves *3* and *4*), are shown in Fig. 1.

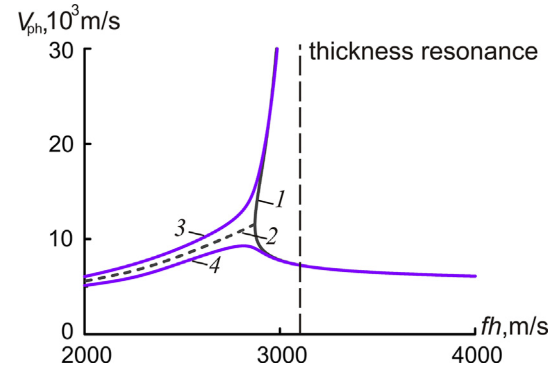

Fig. 1. The phase velocities V_{ph} of the S_1 mode of frequency f in free (*1*, *2*) and water-submerged (*3*, *4*) aluminum plates of thickness h.

The calculations are performed by a method similar to that described in Ref. [21]. The curve *1* is given by the real root of the secular equation and it corresponds to propagating waves, i.e., waves with nonzero time-averaged energy flux. The curve *2* is given by the complex root of the secular equation and it corresponds to nonpropagating waves, i.e., evanescent waves with zero time-averaged energy transfer. The existence range of the backward waves in the free plate is determined by the upper part of the curve *1* located between the thickness resonance frequency and the turning point of the curve (the common point of curves *1* and *2*). The liquid loading results in the hybridization of the real branch *1* and the complex branch *2* and the splitting of the previously continuous curve *1* into two separate complex branches *3* and *4*. The branch *3* belongs to the backward wave, the branch *4* to the forward wave. These hybrid branches have opposite signs of the decay constants α (Fig. 2).

Besides, the attenuation of backward leaky Lamb waves is significantly greater (about ten times for aluminum/water combination) than for the forward ones. The high attenuation can be avoided by choosing media with higher contrast of acoustic impedances like aluminum in liquid helium (Fig. 2).

3. Zero energy flux for backward leaky waves

For further study of the energy properties, Eq. (1) is transformed to a quadratic form [17] by multiplying it by the complex-conjugated vector v_i^* and the complex-conjugate of Eq. (1) by v_i, and then adding these equations by making use of the Hook's law. The result is

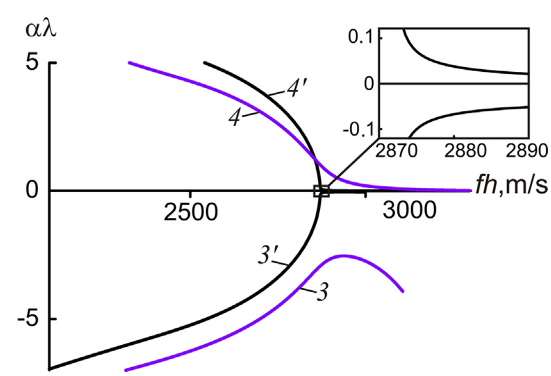

Fig. 2. The normalized leakage (product of the attenuation constant α and the wavelength λ) for the backward and forward modes of aluminum plate in water (*3*, *4*) and liquid helium (*3'*, *4'*).