Ultrasonics 77 (2017) 152–159

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Damage detection of fatigue cracks under nonlinear boundary condition using subharmonic resonance

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ARTICLE INFO

Article history:
Received 1 July 2016
Received in revised form 19 January 2017
Accepted 1 February 2017
Available online 4 February 2017

Keywords:
Fatigue cracks
Subharmonic
Multiple scales method
Piezoelectric transducers
Boundary conditions

ABSTRACT

In recent years, the nonlinear ultrasonic technique has been widely utilized for detecting fatigue crack, one of the most common forms of damage. However, one of limitations associated with this technique is that nonlinearities can be produced not only by damage but also by various intrinsic effects such as boundary conditions. The objective of this paper is to demonstrate the application of a nonlinear ultrasonic subharmonic method for detecting fatigue cracks with nonlinear boundary conditions. The fatigue crack was qualitatively modeled as two elastic, frictionless half spaces that enter into contact during vibration and where the contact obeys the basic Hertz contact law. The nonlinear ordinary differential equation drawn from the developed model was solved with the method of multiple scales. The threshold of subharmonic generation was studied. Different threshold behaviors between the nonlinear boundary condition and the fatigue crack were found that can be used to distinguish the source of nonlinear subharmonic features. To evaluate the proposed method, experiments using an aluminum plate with a fatigue crack were conducted to quantitatively verify the subharmonic resonance range. Two surface-bonded piezoelectric transducers were used to generate and receive ultrasonic wave signals. The experimental results demonstrated that the subharmonic component of the sensing signal could be used to detect the fatigue crack and further to distinguish it from inherent nonlinear boundary conditions.

1. Introduction

Evaluation of structural changes in materials and constructions and monitoring their ultimate strength and endurance in operation are essential for many industrial structural integrity problems. The most powerful nondestructive way of evaluating material degradation is the ultrasonic method because the characteristics of ultrasonic wave propagation are directly related to the properties of the material. The traditional ultrasonic nondestructive damage testing (NDT) is based on a linear theory and normally relies on measuring some particular parameter (sound velocity, attenuation, transmission, and reflection coefficients) of the propagating wave to determine the elastic properties of a material or to detect defects [1]. Recent research has shown interest on using full wavefield data acquired by scanning laser Doppler vibrometer to imaging crack growing in structures [2,3]. Ultrasonic methods based on linear wave scattering are efficient for detecting gross defects and characterizing material elasticity but are less sensitive to microcracks or closed cracks. Elastic waves that propagate through a fatigue crack will not be obviously changed in phase or amplitude, making it difficult to evaluate the imperfect interface using the conventional ultrasonic method based on linear elasticity. To improve the monitoring capability of safety-critical structural components, nonlinear ultrasonic methods have been proposed. If the excitation amplitude is sufficient, closed cracks can behave nonlinearly because of contact dynamics occurring between the faces of the crack. This effect, called contact acoustic nonlinearity (CAN), is assumed derive from the lack of stiffness symmetry for near-surface strain across the interface [4]. Importantly, interface stiffness depends on the contact condition, and becomes a source of nonlinearity in wave propagation. Using the nonlinear behavior of these defects, nonlinear ultrasonic techniques such as nonlinear resonance ultrasound spectroscopy [5–7], higher harmonic generation [8–12], and nonlinear wave modulation spectroscopy [13,14] have been shown to be sensitive to microcracks or closed cracks.

It seems that harmonic generation is a promising technique for the detection of fatigue cracks. Its extreme sensitivity is a benefit for detection, but can also result in high variability in some measurements. For example, electronic equipment will also create nonlinearity in measurement, because the signal directly from a function generator already carries inherent higher harmonics due

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http://dx.doi.org/10.1016/j.ultras.2017.02.001

to the nonlinear electronic system [15]. Such a situation is inevitable and can cause great difficulty and even false alarms in practical applications of higher harmonic techniques. Subharmonics, on the other hand, cannot be introduced by electronic equipment and their generation requires specific conditions, a feature that makes them more reliable for detecting fatigue cracks [16]. Yamanaka et al. [17–19] introduced a breathing crack model and developed a subharmonic array experiment for crack detection to evaluate the model. Johnson [20] developed a single degree of freedom mass-spring oscillator with bilinear stiffness and adopted a finite element model of a cracked rod to verify the theoretical exciting frequency condition of a subharmonic. Mahmoodi et al. [21] used the method of multiple scales to derive the subharmonic resonance appearing in nonlinear flexural vibrations of a piezoelectrically actuated micro-cantilever, and experimental examination verified the analytical results.

Although analytical models of fatigue cracks have been developed, the models in [17–20] have been limited to analyses with free boundary conditions. Another challenge is that harmonics and subharmonics may also stem from boundary conditions, which however are not a kind of structural damage. It is then difficult to figure out whether the harmonics stem from structural damage or from the intrinsic nonlinear boundary conditions. Aymerich et al. demonstrated the importance of the effect of boundary conditions when nonlinear acoustics is used for impact damage detection [22,23]. Qiu et al. proposed an online updating Gaussian Mixture Model (GMM)-based damage evaluation method to improve damage evaluation reliability under time-varying conditions [24]. Using the method of multiple scales, Ogam et al. demonstrated that the Hertz contact force in parallel to the fractional-order derivative dashpot accounts for a contact friction boundary condition [25].

To date, little research into distinguishing defect-induced nonlinearities from boundary nonlinearities has been reported. However, subharmonics generally exhibit a threshold behavior whereby they can only occur when certain conditions on the input (amplitude and frequency) are satisfied. The specific excitation conditions make it possible to distinguish damage-related from non-damage-related nonlinear sources (boundary conditions).

This paper aims to demonstrate the application of a nonlinear ultrasonic subharmonic method for detecting fatigue cracks with nonlinear boundary conditions. A single degree of freedom (SDOF) mechanical vibration model approach made up of a spring in parallel with a nonlinear restoring force attached to a mass was developed to simulate nonlinearity caused by fatigue cracks and nonlinear boundary conditions, especially clamping conditions. Both fatigue cracks and clamping conditions were modeled by supposing that the two surfaces enter into contact during vibration and that the contact obeys the basic Hertz contact law. The equation governing the motion of the structure was then found to display quadratic and cubic nonlinearity. For the threshold phenomenon of subharmonics, the method of multiple scales was used to obtain an analytical expression of a SDOF under the subharmonic resonance excitation. Then experiments on an aluminum plate with a fatigue crack were conducted to quantitatively verify the subharmonic resonance range and the different threshold behaviors of nonlinear boundary conditions and fatigue cracks. The results demonstrated that subharmonic methods have the potential to differentiate damage from the effects of nonlinear boundary conditions.

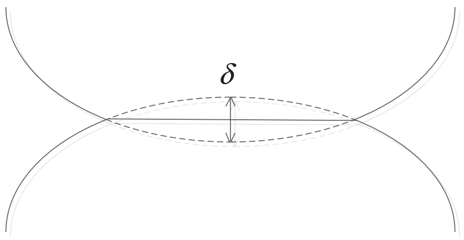

Fig. 1. Deformation of two spheres in contact.

2. Theoretical analysis

2.1. Theoretical model

Crack surfaces are usually rough, and thus surface asperities rather than a whole surface support the load when an interface between two solids comes into contact. More importantly, asperities are deformable, elastically or plastically. Due to the changes of contact the mechanical response of the contact interface involves certain nonlinear behavior. Hence, models of cracks based on the theory of contact mechanics are presented in this section, shown in Fig. 1. In such models, a crack is considered as a contact of two elastic, frictionless half-spheres. When the two spheres are in contact, the restoring contact force between spaces according to Hertz theory is given by [26]

$$y(\delta) = \kappa\delta^{\chi} \tag{1}$$

where δ is the indentation depth, κ is a constant dependent on the elastic and geometric properties of contact surfaces. The exponent for planar contacts of various materials is 1.6–3.3. The relation $y \propto \delta^{\chi}$ holds not only for spheres but also for other bodies of finite dimensions [27]. Nonlinear boundary conditions such as the clamping condition can equally be modeled as two elastic surfaces in contact [28]. However, the contact pressure and the asperities related to the parameters in Hertz theory differ between fatigue cracks and boundary conditions, an observation that is the basis for the distinction between nonlinearities due to damage and nonlinearities due to other effects such as boundary conditions.

The objective of this paper is to demonstrate the application of a nonlinear ultrasonic subharmonic method for detecting fatigue cracks with nonlinear boundary conditions rather than accurate modeling from the physical point of view. To illustrate the feasibility of the subharmonic resonance and ascertain its threshold condition, we assume that the damaged area can be identified as a nonlinear oscillator with a SDOF, as shown in Fig. 2. The nonlinear oscillator is driven by an external acoustic excitation. The contact force takes the form of the Maclaurin power series expansion around a static point h_0 ($\delta = x - h_0$),

$$\begin{aligned} y(\delta) &= \kappa\delta^{\chi} = \kappa(x-h_0)^{\chi} \\ &= \kappa(-h_0)^{\chi}\left(1 - \chi h_0^{-1}x + \frac{1}{2}(\chi-1)\chi h_0^{-2}x^2 \right. \\ &\qquad \left. - \frac{1}{6}(\chi-2)(\chi-1)\chi h_0^{-3}x^3 + \ldots\right). \end{aligned} \tag{2}$$

The equation of motion for this model is given by:

$$m\ddot{x} + c\dot{x} + kx + y(\delta) = F\cos(\Omega t), \tag{3}$$

where x is absolute displacement of the mass relative to a fixed reference, k and c are the linear elastic stiffness and viscous damping coefficient, respectively. F and Ω are the excitation force amplitude and angular frequency, respectively. To simplify the derivation, it is assumed that $\frac{c}{2m} = \mu$, $\frac{k}{m} = \tilde{\omega}_0^2$, $\frac{k}{m} - \frac{\kappa(-h_0)^{\chi}\chi h_0^{-1}}{m} = \omega_0^2$, $H = \frac{\kappa(-h_0)^{\chi}}{m}$, $f = \frac{F}{m}$, $\frac{\kappa(-h_0)(\chi-1)\chi h_0^{-2}}{2m} = \alpha$, $-\frac{\kappa(-h_0)^{\chi}(\chi-2)(\chi-1)\chi h_0^{-3}}{6m} = \beta$. Where μ is the modified viscous damping coefficient, $\tilde{\omega}_0$ is the initial natural frequency, ω_0 is the modified natural frequency, H is a constant, f is the modified excitation force amplitude. α, β are the coefficient of quadratic and cubic term, respectively. From the simplification it can be seen that the structural natural frequency ω_0 is supposed to be lower