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Acoustic radiation force on a sphere in a progressive and standing zero-order quasi-Bessel-Gauss beam

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ABSTRACT

By means of series expansion theory, the incident quasi-Bessel-Gauss beam is expanded using spherical harmonic functions, and the beam coefficients of the quasi-Bessel-Gauss beam are calculated. According to the theory, the acoustic radiation force function, which is the radiation force per unit energy on a unit cross-sectional surface on a sphere made of diverse materials and immersed in an ideal fluid along the propagation axis of zero-order quasi-Bessel-Gauss progressive and standing beams, is investigated. The acoustic radiation force function is calculated as a function of the spherical radius parameter ka and the half-cone angle β with different beam widths in a progressive and standing zero-order Bessel-Gauss beam. Simulation results indicate that the acoustic radiation forces with different waist radii demonstrate remarkably different features from those found in previous studies. The results are expected to be useful in potential applications such as acoustic tweezers.

1. Introduction

Acoustic radiation forces in non-diffraction beams such as Bessel waves, which are not subject to natural diffractive spreading rules as explained by the Huygens-Fresnel principle, have been a popular topic of research point. Relevant scientific studies in this area have been published by researchers like Mitri [1–4] and Marston [5–7]. The acoustic radiation forces on a sphere in a Gauss progressive and standing wave, which has energy-focusing properties, were investigated by Zhang and Wu [8,9]. In optics, the axicon-based Bessel-Gauss resonator with concave output coupler was presented in 2003 [10]. Hakola demonstrated a simple and compact laser source that directly produces a Bessel-Gauss beam [11]. Altucci experimentally investigated the use of diffraction-free Bessel-Gauss beams to generate low-order harmonics in gas [12]. In acoustics, Ding investigated the theoretical properties of the fundamental and second harmonic components of the Bessel-Gauss beam [13]. Mitri proposed a method based on the Rayleigh-Sommerfeld surface integral to obtain rigorous partial-wave series expansions for the incident field of acoustic spiraling Bessel-Gauss beams [14]. Wang presented an approximate analytical description for Bessel-Gauss beams with a finite aperture [15]. However, no investigation of the acoustic radiation forces on a sphere in a quasi-Bessel-Gauss wave has been published to date [16].

The purpose of this investigation is to combine the results of prior studies to provide a general expression for the acoustic radiation force of zero-order quasi-Bessel-Gauss progressive and standing wave fields on a sphere immersed in a non-viscous fluid. The quasi-Bessel-Gauss wave possesses the non-diffraction advantages of the Bessel wave and the energy-focusing properties of the Gauss wave, and unusual phenomena emerge in the simulation. These results could be helpful in understanding the influence of the mechanism for Bessel and Gauss components on the acoustic radiation force of a quasi-Bessel-Gauss wave. Moreover, it could be significant for other potential applications in particle manipulation and entrapment.

2. Theory

The Geometry of the problem for progressive quasi-Bessel-Gauss wave is exhibited in Fig. 1. In the spherical coordinates system, r denotes the distance from the observation point to the center of the sphere, and θ the scattering angle relative to the axis. The standard Bessel-Gauss optical beam, which is a complex solution of the paraxial wave equation, was thoroughly discussed in detail by Bouchal [17], Gori [18], and others. The magnitude and beam width

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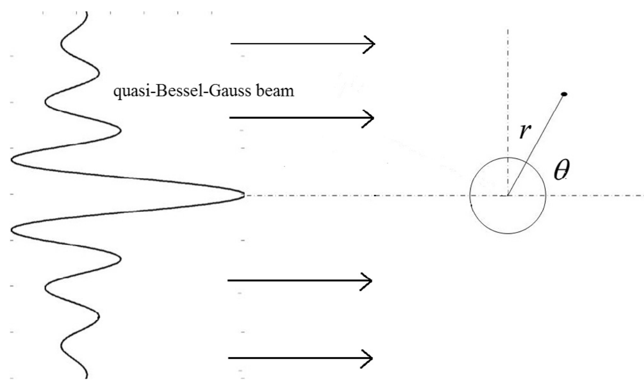

Fig. 1. Geometry of the problem.

of the standard Bessel-Gauss beam varies with distance from the focal region. Neglecting the viscosity and dissipation of an infinite-extent ideal fluid, in which a homogeneous sphere is immersed along the axis of a zero-order quasi-Bessel-Gauss beam, the incident velocity potential of a zero-order quasi-Bessel-Gauss progressive wave, which is a local approximation of a Bessel-Gauss beam very near the focal region at z = 0, can be expressed as:

$$\phi_i = \phi_0 J_0(k_r r_\perp) e^{-(k_r r_\perp / kW_0)^2} e^{i(k_z z - \omega t)}, \tag{1}$$

where ϕ_0 is the amplitude of the velocity potential, k is the wave number, $k_z = kcos\beta$ and $k_r = ksin\beta$ denote the axial and radial wave numbers, and β is the half-cone angle of the plane wave components. $z = rcos\theta$ and $r_\perp = rsin\theta$ represent the axial and radial lengths, ω is the angular frequency of the wave, W_0 is the width of the Gauss beam, and J_0 is a zero-order cylindrical Bessel function, which stands for the zero-order Bessel component of the beam. Although the Gauss component is characterized by the beam width W_0, when W_0 becomes infinitely large, the quasi-Bessel-Gauss beam degenerates to a basic Bessel beam.

It is essential to note that the calculation results in this study are subject to the following constraints. The beam width parameter kW_0 should greatly exceed 1, and the approximation in Eq. (1) assumes that $z/b \ll 1$, where $b = k(W_0)^2/2$. Therefore, the constraint $2kz \ll (kW_0)^2$ should be respected to avoid phase error.

2.1. Beam coefficients calculation

By means of series expansion theory, the incident velocity potential of a zero-order quasi-Bessel-Gauss progressive wave can be expanded with orthogonal spherical properties and finite series properties as:

$$\begin{aligned}\phi_i &= \phi_0 e^{-i\omega t} J_0(kr\sin\theta\sin\beta) e^{-(kr\sin\theta\sin\beta/kW_0)^2} e^{ikr\cos\theta\cos\beta} \\ &= \phi_0 e^{-i\omega t} \sum_{n=0}^{\infty} (2n+1) i^n j_n(kr) P_n(\cos\theta) C_n P_n(\cos\beta),\end{aligned} \tag{2}$$

where j_n and P_n separately designate the n-th order spherical Bessel function and the Legendre function $\phi_0 e^{-i\omega t}$ can be neglected in the calculations for convenience. The coefficients C_n for odd and even components can be independently calculated using the special value method involving expansion with spherical harmonic functions, which is also called finite series expansion and was originally used to calculate the coefficients of Gauss beams in electromagnetic fields [19]. The method can be used to compute the approximate coefficients C_n for quasi-Bessel-Gauss beams using Eq. (2). If $s = 1/kW_0$ is defined as the beam width parameter, and supposing n is even, n can be replaced by $2l$. The coefficient results C_n for n even can be obtained as:

$$C_{2l} = (-1)^l \sum_{j=0}^{l} 2^{(2l-2j)} \Gamma(1/2 + 2l - j) B_{2l-2j} / (\sqrt{\pi} P_{2l}^2(0) j!), \tag{3}$$

Similarly, the coefficient results C_n for n odd is obtained

$$\begin{aligned}C_{2l+1} = (-1)^l \sum_{j=0}^{l} 2^{(2l-2j+1)} \Gamma(3/2 + 2l \\ - j) B_{2l-2j+1} / (\sqrt{\pi} P_{2l+1}'^2(0) j!),\end{aligned} \tag{4}$$

where Γ is the gamma function, $P_{2l}(0)$ represents the Legendre polynomials of order 2l, and $P'_{2l+1}(0)$ represents the Legendre polynomials' differential of order $2l + 1$. Their values with argument 0 are shown in Appendix C. Parameter B is an intermediate value that can be calculated for n both even and odd as

$$B_{2l+1} = B_{2l} = \sum_{j=0}^{l} (-1)^l s^{2l-2j} / (j!^2 4^j (l-j)!). \tag{5}$$

The detailed calculation procedure is given in Appendix D. To validate the series expansion method, the incident zero-order quasi-Bessel-Gauss beam is respectively calculated by exact solution using Eq. (1) and by the finite series expansion method for different beam widths. The results are shown in Figs. 2a–2c. The calculation results were found to be nearly identical, which proves the correctness of this method.

2.2. Acoustic scattering by sphere and acoustic radiation force

The scattering velocity potential produced by the sphere can be expressed as:

$$\phi_s = \phi_0 e^{-i\omega t} \sum_{n=0}^{\infty} (2n+1) i^n S_n h_n^{(1)}(kr) P_n(\cos\theta) C_n P_n(\cos\beta), \tag{6}$$

where $h_n^{(1)}$ denotes the spherical Hankel function of first order. The scattering coefficients $S_n = \alpha_n + i\beta_n$ are determined by the boundary condition at the surface of the sphere, and they are also relevant to the material properties of the sphere and the fluid in which the sphere is immersed. α_n and β_n are the real and imaginary parts of the scattering coefficients S_n for plane waves.The total velocity potential distribution outside the sphere is regarded as a combination of the scattering and incident wave fields and is given by:

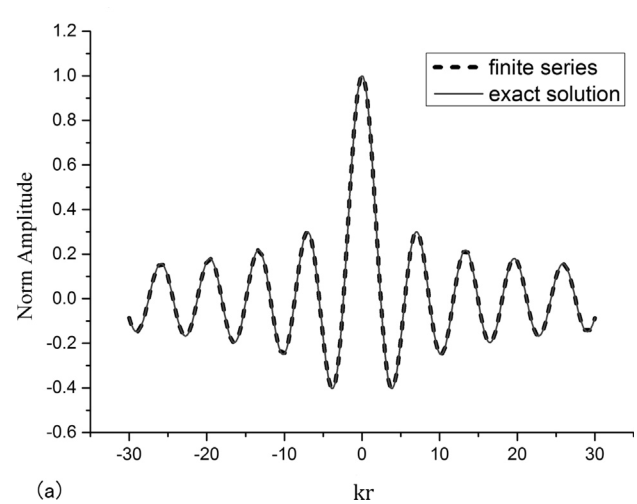

Fig. 2a. Contrast of the incident zero-order quasi-Bessel-Gauss beam computed by exact solution and finite series expansion as $\theta = \pi/2$ and $\beta = \pi/2$ with beam width $W_0 = \infty$.