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Topological imaging in bounded elastic media

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ABSTRACT

Detecting, imaging and sizing defects in a bounded elastic medium is still a difficult task, especially when access is complex. Adjoint methods simplify the task as they take advantage of prior information such as the geometry and material properties. However, they still reveal a number of important limitations. Artifacts observed on the conventional topological energy image result from wave interactions with the boundaries of the inspected medium. The paper describes a method for addressing these artifacts, which involves forward and adjoint fields specified in terms of the boundary conditions. Modified topological energies are then defined according to the type of analyzed flaw (open slit or inclusion). Comparison of the numerical results with the experimental data confirms the relevance of the approach.

1. Introduction

In the context of non-destructive evaluation (NDE) model-based methods are developed and applied to detect, locate and characterize embedded defects in an inspected structure. Among these methods there is a range that uses an ultrasonic array transducer and the full matrix capture (FMC) acquisition technique. Assuming a constant wave celerity of the medium, post-process is then performed on this data to obtain either a traditional B-scan or a more advanced focused image based on the total focusing method (TFM) [1]. The latter technique is used to synthetically focus the energy back to every point in a region of interest.

Techniques based on the conventional time-reversal process [2] or iterative approach such as DORT [3] have also been successfully applied for inspecting elastic media, provided that defects are strong scatterers.

Topological imaging [4–6] is a recent NDE method, related to time reversal and adjoint methods which makes it possible to image an inspected medium. This method was originally developed from optimization approaches used to solve inverse acoustic or elastic problems in the field of geophysics [7–9] and mechanical structure optimization [10–13]. The generic problem is to find the unknown parameters distribution that best fits the actual measured data. Thus the approach is to iteratively construct a parameter distribution, which provides synthetic data in agreement with the measured data. A cost function which quantifies the goodness of fit between predicted and observed data is minimized. This process requires computating of the cost-function gradient (i.e. the Fréchet derivative). Tarantola [8] and Norton [14] demonstrate that the Fréchet derivative can be computed by involving the interaction between two numerical wave fields: forward and adjoint. The theory of the adjoint method for Fréchet derivative computations has been developed under the Born approximation in a fluid medium [8,14] and in a solid medium [15,16].

In the Born approximation, the total field inside the scattering object is replaced by the incident field present in the surrounding medium that is assumed infinite and homogeneous. This approximation is effective when the contrast is small and as long as the size of the object is not too large relative to the wavelength. The domain of validity of the Born approximation is disccused discussed in Section 8.10.1 of Chew [17]. Noting k the wave number, q the contrast function and $|D|$ the d-dimensional volume of the scatterer ($d = 2$ or 3), two limits are identified:

- in the low-frequency, the long-wavelength limit (i.e. if $k|D|^{\frac{1}{d}} \ll 1$)

$$k^2|D|^{\frac{2}{d}}\max_D|q| \ll 1$$

- in the high-frequency, the short-wavelength limit (i.e. if $k|D|^{\frac{1}{d}} \gg 1$)

$$k|D|^{\frac{1}{d}}\max_D|q| \ll 1$$

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Thus, in tri-dimensional space, under low-frequency assumption, $k|D|^{\frac{1}{d}} \approx 0.1$, contrast variations up to $q = 10^2$ are tractable. Under high-frequency assumption, $k|D|^{\frac{1}{d}} \approx 0.1$, the constraint $q < 10^{-2}$ significantly limits the Born approximation applicability. Significant improvements in the Born approximation have been achieved by employing the extended Born approximation (EBA) [18,19]. Under acceptable computing cost, this method allows to obtain accurate simulations for a larger range of material contrast and object sizes because of the consideration of multiple scattering effects. However, the accuracy of the EBA degrades when the scatterers are close to the source region or else, when the field exhibits significant spatial variations within the scatterer. Therefore, high-order variants of the Born approximation that make use of series expansion of the field have been proposed. However, their iterative implementations require higher computing cost, and eventually are proned to diverge. In this paper, we investigate a non-iterative method for flaw detection in a bounded domain, which does not require a priori knowledge of the flaw shape and location and which is able to localize it using a limited aperture. It has been observed in many studies, that the topological derivative depends on the frequency of the excitation waves. For instance, in scattering problems for the Helmoltz equation in unbounded media, the topological derivative selects points inside flaws when low/medium frequencies are considered, while for high frequencies, the selected points are localized in a small region close to the boundaries [20–22]. The question why the high-frequency inverse scattering by topological sensitivity may work has been studied in details in [23]. It is shown that at higher frequencies, the topological sensitivity is asymptotically dominated by the near-boundary terms. When a reduced number of measurements is available, low frequencies are generally preferred. In that case, under the Born approximation assumption, it has been shown that the contrast function $q$can theoretically be recovered up to a characteristic length scale of half the wavelength (low-pass filtering). Indeed, the full-aperture topological derivative is proportional to [24] $|D|k^4[q * \psi^2](\mathbf{x})$, where $\psi(\mathbf{x}) = 4\pi \sin_c(k|\mathbf{x}|)$ is analytic everywhere and $*$ is the convolution product. In consequence, it is not the contrast object q which can be recovered but only its filtered vesrion $q * \psi^2$. Thus, unless specified, the object contrast considered (in the following) is the filtered one, which is therefore analytic. Furthermore, it has been shown that the topological derivative can provide satisfactory defect reconstructions beyond the Born limits [24]. These theoretical results are derived in the case of infinite media and using time-harmonic waves. But, topological derivatives can also be used for multi-frequency scattered data, with a reduced number of aligned emitters and receivers [25,26]. Oppositely to infinite domains, an interesting study is described in [27] in case of bounded media, where it is shown that, even under large aperture, single-frequency topological derivatives do not necessarily reveal the defect support. However, considering a composition of non-evenly distributed frequencies and associating their weighted topological derivatives leads to relevant reconstructions when using medium frequencies ($0.3 \leqslant k|D|^{\frac{1}{d}} \leqslant 7.5$) and for defect placed away from the boundaries. This approach involves calculation of several topological derivatives which requires high computational cost. Besides, it is clearly shown in that study that the quality of the image degrades as the number of emitters/receivers reduces. A way to compensate the spatial constraint is to enrich the spectral content of the interrogating waves. In the present study, accurate defect (holes or open cracks) imaging is achieved using a cost efficient approach that involves computation of a single time-domain (broad-band waves) topological derivative. In practice, the evaluation of the Fréchet derivative firsts consists in insonifying and measuring with a linear transducer array, the broad-band responses of the reference and the inspected media.

The residue is calculated as the difference between these two ultrasonic responses. Two wave fields are then numerically computed in the time domain: the forward and the adjoint fields. These fields satisfy the elastic wave propagation in a numerical model of the reference medium. To obtain the forward field, the source signal (*initial conditions*) is defined by simulating the experimentally transmitted (broad-band) waveform. Concerning the computed adjoint field, sources, located at the receiver locations, transmit the time-reversed residue (*adjoint initial conditions*). Thus, the topological energy image is computed from a dynamic windowing of both wave fields. This topological energy method (TEM) has been investigated with acoustic and elastic waves [28,5,29], with dispersive waves [6] and with anisotropic waves [30]. The method successfully highlights defects considered as relevant distinctions when compared with the reference model.

This adjoint method, based on Fréchet derivative analysis, has been introduced under the Born approximation. Difficulties to apply this process in a bounded medium also arise because multiple reflections of the forward and adjoint fields occur and may coexist at different times and locations during the whole duration of the acquisition process.

This paper presents a new approach for addressing multiple reflections. It focuses on the application of the TEM in a bounded medium by selecting the relevant information. Section 2 presents a theoretical basis for the TEM. Two modified topological energies are defined which make it possible to select distinct contents of information. We show that by modifying boundary conditions in the numerical model we can remove unwanted artifacts due to reflections on the edges and thus enhance performance. Section 3 presents numerical and experimental topological energy images of representative defects such as a hole or inclined slit. The results are then compared and discussed in Section 4 and the conclusions are presented in Section 5.

2. Topological Energy Method (TEM)

2.1. Theoretical background

Let us consider two isotropic homogeneous bounded media. The first is unperturbed (reference medium) while the second has a perturbation (inspected medium). Both of them are insonified by a linear transducer array. Each point source at location $\mathbf{x}_s$ transmits a spherical wave to both media. Signals are measured on N receiving points $\{\mathbf{x}_r, r = 1, \ldots, N\}$ after wave propagation into the media. The residue at the receivers $\Delta u(\mathbf{x}_r, t)$ is then computed as

$$\Delta u(\mathbf{x}_r, t) = u_0(\mathbf{x}_r, t) - u_{obs}(\mathbf{x}_r, t), \tag{1}$$

where u_0 and u_{obs} are the normal components of displacement measured in the unperturbed and inspected medium respectively. The residue is an ultrasonic signature of the perturbation at the receiver locations and it depends on the parameter filtered distribution $n(\mathbf{x})$ at any point $\mathbf{x}$ of the perturbed medium. The cost function denoted χ is defined as

$$\chi[n(\mathbf{x})] = \frac{1}{2}\sum_{r=1}^{N}\int_0^T ||\mathbf{u}_0(\mathbf{x}_r, t) - \mathbf{u}_{obs}(\mathbf{x}_r, t)||^2 dt, \tag{2}$$

where T is the acquisition duration.

The TEM is based on the calculation of this cost function and its topological asymptotic expansion [10]:

$$\chi[n(\mathbf{x}) + \delta n(\mathbf{x})] = \chi[n(\mathbf{x})] + \delta n(\mathbf{x}) g(\mathbf{x}) + \mathcal{O}(\delta n(\mathbf{x})), \tag{3}$$

where $\delta n(\mathbf{x})$ represents an infinitesimal variation of the medium's properties. For localizing a change in the medium, the TEM