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Short communication

Statistics associated with the scattering of ultrasound from microstructure

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ABSTRACT

The spatial statistics of an ensemble of waveforms containing ultrasonic scattering from microstructure are investigated. The standard deviation of the waveforms is of primary interest, because it is related to the maximum scattering amplitudes in the extreme value statistics theory. Further statistical measures are employed to define theoretical confidence bounds, which bound the experimentally calculated maximum amplitude when a finite number of waveforms are included in the ensemble. These statistical measures are applied in conjunction with a previously developed ultrasonic backscatter model. It is validated through ultrasonic scattering measurements performed on a stainless-steel pipe sample. These considerations are important for forward models related to the probability of detection (POD) of defects and inverse models used for characterization of polycrystalline microstructures.

1. Introduction

In metals, acoustic impedance mismatches located at the grain boundaries between adjacent grains exist. These interfaces lead to the scattering of ultrasound, which is often called grain noise when observed within ultrasonic waveforms [1]. Typically, grain noise is undesirable because it can hide reflected echoes associated with defects and can cause them to go undetected. Thus, forward models that quantify the ultrasonic scattering in polycrystalline materials, often known as the Figure-of-Merit (FOM), is an important input to model-assisted probability of detection (MAPOD) [2]. Conversely, grain noise is desirable when using it as a tool for microstructure characterization. In this case, the grain noise is known as ultrasonic backscattering and has been related to: (1) a variety of morphological properties of grains [3–6], and (2) the elastic properties of the polycrystals and grains [7,8].

In this short communication, we highlight the experimental measurement procedure employed in recent publications by Turner and co-workers [8–10]. These measurements make use of a statistical variance measure on a collection of spatially independent scattered waveforms. The experimental variance was theoretically represented as the singly-scattered response (SSR) [10]. The SSR assumes that the ultrasonic waves scatter only once from the microstructure. The present work introduces concepts from extreme value statistics [11] into ultrasonic scattering from polycrystalline microstructures by considering any finite number of waveforms belonging to the ensemble. The modified model is verified by scattering measurements performed on a stainless-steel pipe sample.

2. Theory

The variance of an ensemble of collected ultrasonic waveforms is [9]

$$\Phi^{\exp}(t) = \frac{1}{N}\sum_{i=1}^{N} V_i^2(t) - \left(\frac{1}{N}\sum_{i=1}^{N} V_i(t)\right)^2, \qquad (1)$$

where i denotes the i-th waveform in the ensemble containing a total of N normally distributed waveforms and $V_i(t)$ is the time-dependent amplitude (typically a voltage) of the i-th waveform. Notice that the $V_i(t)$ are assumed to be distributed in a Gaussian manner at all depths, even in the focal zone, although this need not be the case in general [12]. The variance is denoted with the superscript *exp* to emphasize that it is an experimentally measureable parameter. The spatial variance was modeled in the previous work of Turner and co-workers [8–10]. The theoretical longitudinal to longitudinal SSR model for a pulse/echo transducer configuration (immersion) at normal incidence to the sample is written as [8],

$$\Phi^{\text{theory}}(t) = \Phi^{0}_{LL}\tilde{\eta}_{LL}\Xi^{\cdots\hat{\mathbf{p}}\hat{\mathbf{p}}\hat{\mathbf{s}}\hat{\mathbf{s}}}_{\cdots\hat{\mathbf{p}}\hat{\mathbf{p}}\hat{\mathbf{s}}\hat{\mathbf{s}}}\psi_{LL}, \qquad (2)$$

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where Φ^0_{LL} is a constant related to the experiment calibration. Here, $\tilde{\eta}_{LL}$ and $\Xi^{\cdots\hat{p}\hat{p}\hat{s}\hat{s}}_{\cdots\hat{p}\hat{p}\hat{s}\hat{s}}$ represent geometric and elastic properties of the microstructure, respectively, while ψ_{LL} is the field distribution function describing the transducer beam and the input pulse. Explicit parameter definitions can be located in Eq. (30) of Ref. [9]. Strong agreement between $\Phi^{\exp}(t)$ and $\Phi^{\text{theory}}(t)$ has been established previously [9].

Next, we seek the relation between the maximum amplitudes present in the ensemble and the spatial standard deviation. Strong assumptions are used here: (1) the polycrystalline materials should be strictly statistically homogeneous; (2) the samples are well polished and their two surfaces are highly parallel; (3) measurement system effects (e.g. electromagnetic interference, averaging time, scanning speed, etc.) can be neglected; (4) the separation between two consecutive transducer positions should be large enough that the two backscattered signals are fully uncorrelated; (5) there are no vertical offsets to the baseline signal. All of these assumptions are used to ensure that $\langle V(t)\rangle = \sum_{i=1}^{N}[V_i(t)/N] \equiv 0$, and all the waveforms are independent and identically distributed (IID). Thus, the extreme value statistics [11] can be introduced to describe the relationship between the maximum amplitudes in the ensemble $A^{\exp}_{\max}(t) = \max(|V_i(t)|)$ and the spatial standard deviation $\Sigma^{\exp}(t) = \sqrt{\Phi^{\exp}(t)}$.

In statistics theory, $|V_i(t)|$ obey the half normal distribution when $V_i(t)$ obey the normal distribution with zero-mean and standard deviation $\Sigma(t)$. Let $A_{\max}(t)$ denote the extreme value of $|V_i(t)|, i = 1, 2, \ldots, N$, and $A(t)$ denotes the possible value of $A_{\max}(t)$. Hence, $A_{\max}(t)$ obeys a Gumbel distribution, whose cumulative distribution function is [11]

$$\Pr\{A_{\max}(t) \leqslant A(t)\} = F_N(A(t)) = \exp\left\{-\exp\left[-\frac{A(t)-b_N(t)}{a_N(t)}\right]\right\}, \tag{3}$$

and the corresponding probability density function can be written as [11]

$$f_N(A(t)) = \frac{1}{a_N(t)}\exp\left\{-\frac{A(t)-b_N(t)}{a_N(t)} - \exp\left[-\frac{A(t)-b_N(t)}{a_N(t)}\right]\right\}, \tag{4}$$

where the normalized constants $a_N(t)$ and $b_N(t)$ can be defined as

$$a_N(t) = \frac{\Sigma(t)}{\sqrt{2\ln N}}, b_N(t) = \left[\sqrt{2\ln N} - \frac{\ln\ln N + \ln\pi}{2\sqrt{2\ln N}}\right]\Sigma(t). \tag{5}$$

Based on the Gumbel distribution, the mathematical expectation or mean value and the confidence bounds of $A(t)$ can be given [11].

In fact, the value of $A_{\max}(t)$ is approximately equal to the mathematical expectation of $A(t)$. Since $\langle A(t)\rangle = b_N(t) + a_N(t)\gamma$ is given by the Gumbel distribution [11], $A_{\max}(t)$ can be estimated as

$$A_{\max}(t) \approx \langle A(t)\rangle = \left[\sqrt{2\ln N} - (\ln\ln N + \ln\pi - 2\gamma)/(2\sqrt{2\ln N})\right]\Sigma(t). \tag{6}$$

where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. $\Sigma(t)$ in Eq. (6) can be experimentally measured and this may be appropriate if the local component curvature is constant, the underlying microstructure is uniform with respect to lateral transducer movements, and a large enough scanning area is available to capture a sufficient number of independent backscattered signals. When this is not the case it may be more prudent to rely on the model estimated $\Sigma^{\text{theory}}(t)$. As strong agreement between $\Sigma^{\text{theory}}(t)$ and $\Sigma^{\exp}(t)$ has been established in previous work [9], utilization of $\Sigma^{\text{theory}}(t)$ in Eq. (6) allows theoretical estimates of the maximum possible level of grain noise, namely $A^{\text{theory}}_{\max}(t)$.

Further, the quantile function of the Gumbel distribution is given by $Q(p) = b_N - a_N\ln(-\ln p)$ [11]. Thus, the upper and lower confidence bounds on estimating $A^{\text{theory}}_{\max}(t)$ can be defined as

$$U^{\text{theory}}(t) = \left[\sqrt{2\ln N} - \frac{\ln\ln N + \ln\pi + 2\ln[-\ln(1-(1-\alpha)/2)]}{2\sqrt{2\ln N}}\right]\Sigma^{\text{theory}}(t), \tag{7}$$

$$L^{\text{theory}}(t) = \left[\sqrt{2\ln N} - \frac{\ln\ln N + \ln\pi + 2\ln[-\ln((1-\alpha)/2)]}{2\sqrt{2\ln N}}\right]\Sigma^{\text{theory}}(t), \tag{8}$$

where α is confidence level. In practice, the upper bound can be used to establish amplitude thresholds to be triggered by an echo caused by a flaw.

3. Experiments and results

To illustrate the application of the proposed methodology, measurements were conducted with a section of 304 stainless steel pipe sample (outer radius 108.5 mm, inner radius 88.0 mm, height 100.2 mm). The sample was well polished with 1000 grit sandpaper. Optical microscopy revealed an equiaxed microstructure having a mean grain diameter of 49.4 ± 3.5 μm via ASTM standard E112. Normal incidence pulse/echo measurements were conducted using a JSR DPR-300 pulser/receiver, a 10 MHz focused transducer (2-in. focal length, 0.5-in. element diameter), a 1 GHz DAQ card and a computer-controlled micropositioning system. The spot size of this focused transducer was ~2.3 mm (much larger than the mean grain diameter of 49.4 μm), which means the beam volume includes a sufficient number of grains such that backscatter data are expected to have a normal distribution. The couplant used was water. The single crystal elastic constants and density of the stainless steel were assumed as: c_{11} = 204.6 GPa, c_{12} = 137.7 GPa, c_{44} = 126.2 GPa, and ρ = 7930 kg/m^3 [13]. The values of the velocities and attenuations for the water and sample were measured as: c_f = 998 m/s, c_L = 5739 m/s, α_f = 2.14 Np/m, α_L = 3.86 Np/m. The water path used was z_f = 19.6 mm, which corresponds to a material path of 8.0 mm using $z_S = (F - z_f)c_f/c_L$. The resolution or step-size in the radial direction was 0.8 mm and 0.6° in the hoop direction. Possible correlations between waveforms are assumed negligible because the resolution is large enough.

A sequence of measurements was performed which collected N = 11, 186, and 4800 waveforms from the top planar face of the sample, respectively. The spatial correlation coefficient is checked to be 0.18 ± 0.16 by a time gate from 28 through 33 μs to meet the IID condition. Also, the kurtosis is checked to be 3.14 ± 0.33 with the same gate, so the backscatter data can be regarded as approximately normal distributed. The experimental results, theoretical model, and bounds for the three cases are shown in Fig. 1. These results reveal the following points. (1) $A^{\exp}_{\max}(t)$ increases with a increase of N. (2) $A^{\exp}_{\max}(t)$ agrees well with $A^{\text{theory}}_{\max}(t)$; this highlights the repeatability of the experiments and the independence of N. (3) The bounds work well no matter what N is. The actual probability that $A^{\exp}_{\max}(t)$ lie between the bounds is checked as 98.04% for Fig. 1 (c) on the same gate; this error is induced by the leptokurtosis and non-Gaussian distribution of actual backscatter data.

Therefore, we can estimate the maximum possible grain noise amplitudes as $U^{\text{theory}}(t)$ when the microstructure is determined. Signal amplitudes greater than the upper bound indicate a flaw echo (FE) as shown in Fig. 2. However, the selection strategy of the confidence level should be explored in the future (considering the dependence on N, as Fig. 1 shows).

In addition, the presented model has limitations because there are many assumptions in it: (1) for the real measurements,