Ultrasonics 80 (2017) 34–46

Contents lists available at ScienceDirect

Ultrasonics

journal homepage: www.elsevier.com/locate/ultras

Self-action of propagating and standing Lamb waves in the plates exhibiting hysteretic nonlinearity: Nonlinear zero-group velocity modes

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ARTICLE INFO

Article history:
Received 28 February 2017
Received in revised form 1 April 2017
Accepted 20 April 2017
Available online 25 April 2017

Keywords:
Lamb waves
Nonlinear waves
Hysteretic nonlinearity
Zero group velocity waves
Nonlinear self-action

ABSTRACT

An analytical theory accounting for the influence of hysteretic nonlinearity of micro-inhomogeneous plate material on the Lamb waves near the S_1 zero group velocity point is developed. The theory predicts that the main effect of the hysteretic quadratic nonlinearity consists in the modification of the frequency and the induced absorption of the Lamb modes. The effects of the nonlinear self-action in the propagating and standing Lamb waves are expected to be, respectively, nearly twice and three times stronger than those in the plane propagating acoustic waves. The theory is restricted to the simplest hysteretic nonlinearity, which is influencing only one of the Lamé moduli of the materials. However, possible extensions of the theory to the cases of more general hysteretic nonlinearities are discussed as well as the perspectives of its experimental testing. Applications include nondestructive evaluation of micro-inhomogeneous and cracked plates.

1. Introduction

Lamb waves are the acoustic eigen modes of a mechanical plate [1,2] and, thus, it is natural to evaluate their propagation for determining the acoustic parameters of a material composing the plate, such as linear acoustic waves velocities and absorption, as well as the nonlinear acoustic parameters. In particular the level of the nonlinear phenomena in Lamb waves propagation and interactions can be a sensitive indicator for the level of material degradation and damage caused by various types of fatigue in the plate [3–7]. Among the nonlinear phenomena, taking place in propagation of the Lamb waves, which could be potentially evaluated for characterization of the material, the dominant attention of the researchers was attracted until now to the processes of the harmonics generation in the initially monochromatic propagating Lamb waves [3–14]. The conditions for the efficient generation of the harmonics such as their phase synchronism with the fundamental wave, requiring the matching of their phase velocities to the velocity of the fundamental wave, as well as the "optimization" of the overlap of their in-depth spatial structures with that of the nonlinear stresses induced by the fundamental wave, are currently well understood [8–12] and successfully applied for sensitive testing of material nonlinearity [3–7,10,12–14]. However, the Lamb waves are dispersive and the experimental realization of the efficient interaction between the harmonics in most of the cases requires a careful choice of a fundamental Lamb mode and its frequency for each particular material in order to have synchronism with a generated higher harmonic belonging to a different Lamb mode [3,6–11]. Because the different order Lamb modes have different in-depth spatial structures, the in-depth matching between the nonlinear stresses and the harmonic modes, that should avoid their orthogonality [9,12] and provide efficient energy transfer from the fundamental wave to one of its harmonics, is never perfect. The most studied in the experiments with nonlinear Lamb waves process of the second harmonic generation has an additional drawback: second harmonic is not generated by the anti-symmetrical Lamb modes [8,9,12]. For example, in the plates with elastic nonlinearity the propagation of flexural waves is accompanied by the generation of the odd harmonics only [12,15–17], while the cubic elastic nonlinearity of the material, causing the generation of the harmonics in the anti-symmetrical Lamb waves, is theoretically expected to be, in the same material, weaker than the quadratic nonlinearity causing the second harmonic generation in symmetric Lamb waves [18,19,12].

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http://dx.doi.org/10.1016/j.ultras.2017.04.010
0041-624X/

Recently it has been proposed [20] that the dominant source of the nonlinear transformation of flexural waves in plates in some cases could be not the elastic nonlinearities (both physical and kinematical/geometrical [18,19,21]) but the hysteretic nonlinearities, essential for a large spectrum of the materials from single-crystals with dislocations [22–25] to polycrystalline/microinhomo geous materials and cracked media [26–35]. The lowest order hysteretic nonlinearity of a material is the hysteretic quadratic nonlinearity (HQNL), which contributes to the nonlinear stresses which are quadratic in the acoustic strain amplitude, i.e., similarly to elastic quadratic nonlinearity, but has, otherwise, the symmetry properties of the odd nonlinearity [21,32,36,37]. For example, it initiates the generation of odd harmonics only, both in propagation of compression/dilatation (c/d) and shear waves [32,36,37]. The goal of our research, which results are presented below, was to provide the simplest extension of the theory of flexural waves in the media with HQNL to the case of an arbitrary, in terms of its symmetry and order, Lamb waves. From the view point of a possible role of the hysteretic nonlinearities of material in the propagation of the Lamb waves the most important is the fact that HQNL initiates self-action of the monochromatic waves already in a single nonlinear scattering process, while the elastic quadratic nonlinearity does not [32,36,37,20]. While the most important outcome of a single nonlinear scattering caused by the elastic quadratic nonlinearity is the generation of the second harmonic, for the HQNL these are slowing down of the propagation velocity and induced nonlinear absorption of the fundamental wave [32,36,37,20]. The self-action of the Lamb wave caused by the HQNL is "automatically" synchronous and "matched" in depth of the plate. The nonlinear stresses at the fundamental frequency are propagating with the same phase velocities as the fundamental wave and their spatial in-depth structure nearly perfectly overlaps with that of the fundamental wave. The nonlinear stresses, which are in phase with the fundamental wave cause its deceleration, while out-of-phase nonlinear stresses cause its nonlinear absorption. Only small part of the hysteretic nonlinear stress is generating higher harmonics [36,38]. Another difference from the elastic quadratic nonlinearity is that even in isotropic media the self-action through HQNL exhibit both compression/dilatation (longitudinal) and shear (transverse) partial components of the Lamb wave, while elastic quadratic nonlinearity induces generation of the second harmonic by the c/d partial wave only. Shear partial wave in the media with elastic quadratic nonlinearity does not generate shear second harmonic itself [18,39,40], shear second harmonic appears only as a result of the c/d second harmonic mode conversion in reflection from the plate surfaces. Finally, the nonlinear self-action of the Lamb waves, caused by the hysteretic quadratic nonlinearity, takes place both for the symmetrical and for anti-symmetrical modes.

2. Lamb waves in isotropic materials with hysteretic nonlinearity

As a preliminary for the analysis of Lamb waves in materials possessing HQNL we present necessary information on a general approach for the description of the acoustic waves in such materials. The equation of motion in nonlinear media has the form

$$\rho_0 \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial}{\partial x_j}\left(\sigma_{ij}^L + \sigma_{ij}^{NL}\right),$$

where ρ_0 denotes the density of the material and u_i denotes the i-th component of the particle displacement vector. We will use the classical presentation for the linear elastic part σ_{ij}^L of the strain tensor [2] and we assume that in the nonlinear part σ_{ij}^{NL} of the stress tensor the hysteretic quadratic terms dominate over the traditional elastic quadratic nonlinearity [21,26,30–34]. So, we consider that $\sigma_{ij}^{NL} = \sigma_{ij}^{HQNL}$. The equation of motion can then be presented in the form

$$\left[\frac{\partial^2}{\partial t^2} - c_T^2\Delta - (c_L^2 - c_T^2)grad\, div\right]\vec{u} = \frac{1}{\rho_0}\vec{e}_i F_i = \frac{1}{\rho_0}\vec{e}_i \frac{\partial \sigma_{ij}^{HQNL}}{\partial x_j}, \qquad (1)$$

where $c_{L,T}$ denote the c/d and shear linear acoustic wave velocities respectively, $\vec{e}_i$ are the unit vectors of the rectangular coordinate frame and F_i are the components of nonlinear body forces. In the frame of Eq. (1) we are applying traditional approach to analyze the interaction of the acoustic waves [9,18,19,21] based on the assumption that the acoustic waves are of finite but small amplitude, ensuring that the nonlinear terms in the right-hand-side (r.h.s.) of Eq. (1) are much smaller than the linear terms in its left-hand-side (l.h.s.). In this case the method of the successive approximations can be applied to search the solutions of Eq. (1). It is assumed that the solution in the first approximation, $\vec{u}$, *i.e.*, the solution to the linearized Eq. (1), is known. In the second approximation the solution of Eq. (1) is assumed to have the form $\vec{u} = \vec{u} + \vec{u}_s$, where $\vec{u}_s$ is the acoustic field scattered/generated due to the nonlinear term in the r.h.s. It is assumed that $\vec{u}_s$ is so small that it does not contribute to the r.h.s. in the second approximation. So only $\vec{u}$ should be substituted in the r.h.s. in the second approximation. However $\vec{u}_s$ contributes to the left-hand-side (l.h.s.) of Eq. (1) and, consequently, we derive the inhomogeneous wave equation with known r.h.s., which is, in fact, the nonlinear source of the generated/scattered wave. This mathematical formalism can be applied to the analysis of the acoustic waves generated due to the acoustic nonlinearity only by the primary nonlinear waves $\vec{u}$, and does not take into account neither subsequent interaction of the generated waves $\vec{u}_s$, with the primary waves nor nonlinear interaction between the new nonlinearly generated waves $\vec{u}_s$. It is common to say that the acoustic field can be found by this approach after a single nonlinear scattering (single scattering approximation). For example, in the case of the elastic quadratic nonlinearity a single nonlinear scattering of the initially monochromatic compression/dilatation wave at cyclic frequency ω leads to the generation of its second harmonic at frequency 2ω. Thus the scattered field is distinct from the primary field in its spectral content. In the case of the HQNL, the result of a single nonlinear scattering of the same wave is drastically different. The scattered field contains all odd harmonics of the primary/fundamental frequency and a component at the fundamental frequency itself [21,36,38,41]. Moreover, the acoustic field scattered by HQNL at the fundamental frequency dominates over the field of all higher harmonics [36,38]. Thus, the main result of a single scattering in the case of HQNL is the modification of the primary monochromatic acoustic field, i.e., the self-action of the primary field. This self-action can be physically interpreted in terms of the dependence of the velocity and attenuation (or the real and imaginary part of the elastic modulus, respectively) on the wave amplitude, which is self-induced by the primary wave [21,36,37].

Of significant importance is that this description of the acoustic field self-action in terms of the amplitude-dependent acoustic material parameters is valid also in case of multiple nonlinear scatterings if the other nonlinear processes contributing to single nonlinear scattering are neglected. As a consequence the equations describing the evolution of the acoustic field caused by its self-action can be derived and the variation of the acoustic field with time can be predicted starting from its initial finite-amplitude state and finishing by its complete disappearance caused by the nonlinear absorption [20,36,37]. In the following we will demonstrate how this approach, described above for the case of plane compres-