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Ultrasonic subwavelength phase conjugated mirror with a layer of bubbles

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ABSTRACT

A single layer of gas bubbles in a yield-stress fluid is experimentally shown to behave as a phase-conjugated (PC) mirror with a thickness 250 times smaller than the wavelength (0.14 mm-diameter bubbles for phase-conjugation at 40 kHz). A high amplitude pump wave at frequency 80 kHz interacts with a lower amplitude probe wave centered at 40 kHz. A PC-reflection coefficient of 0.15 is obtained for a 50 kPa pump. A perturbative second-order theory is shown to quantitatively describe the experimental observations.

1. Introduction

A phase-conjugated (PC) mirror has the property of reflecting an incoming beam in the same direction it came from. The reflected beam thus travels back along the same way, as if time had been reversed. This effect has proven to be useful for correcting phase distortion [1]. It was first demonstrated in optics [2–4], but it can also be obtained with acoustic waves, by using a nonlinear interaction between the acoustic field and (i) an electric field [5], (ii) a magnetic field [6–9], or (iii) the acoustic field itself [10–12]. We give here some details about the latter phenomenon, to illustrate how nonlinear interactions may lead to phase-conjugation.

Let us consider three plane waves, as shown in the top part of Fig. 1, propagating in a nonlinear medium. Each wave, characterized by its amplitude A_n ($1 \leqslant n \leqslant 3$), angular frequency ω_n and wavevector $\vec{k}_n$, generates a field at position $\vec{r}$ and time t given by:

$$A_n \cos(\vec{k}_n \cdot \vec{r} - \omega_n t) = \frac{A_n}{2}\left[e^{i\varphi_n(\vec{r},t)} + e^{-i\varphi_n(\vec{r},t)}\right], \qquad (1)$$

where $\varphi_n(\vec{r},t) = \vec{k}_n \cdot \vec{r} - \omega_n t$ is the phase field of wave n. When the waves propagate in a nonlinear medium, the third-order nonlinear response of the medium generates many waves of the form $A_n A_j A_k \exp(i[\pm\varphi_n(\vec{r},t) \pm \varphi_j(\vec{r},t) \pm \varphi_k(\vec{r},t)])$. In the general case, all these nonlinear contributions are not in phase and cancel each others as they propagate. In other words, the dispersion relation of the medium imposes wave vectors that are not compatible with the nonlinear process. However, a case of particular interest is the $\varphi_1 - \varphi_2 - \varphi_3$ combination, which gives a phase

$$\varphi_{+--} = (\vec{k}_1 - [\vec{k}_2 + \vec{k}_3]) \cdot \vec{r} - (\omega_1 - [\omega_2 + \omega_3])t. \qquad (2)$$

If one chooses waves 2 and 3 with the same frequency as wave 1 ($\omega_2 = \omega_3 = \omega_1$) and opposite directions of propagation ($\vec{k}_2 + \vec{k}_3 = \vec{0}$), the phase field of this nonlinear wave becomes

$$\varphi_{+--} = \vec{k}_1 \cdot \vec{r} + \omega_1 t, \qquad (3)$$

i.e. the same phase field as for wave 1, but in which time has been reversed: $-\omega_1 t$ has been turned into $+\omega_1 t$. This is called the phase-conjugated wave. Bottom left part of Fig. 1 shows how this four-wave mixing with two pump waves leads to a phase-conjugated mirror: the probe wave seems to be reflected by the nonlinear medium in a most unconventional way.

This first configuration relies on a third-order nonlinearity of the "mirror", which is expected to yield quite low intensity fields. However, it benefits from the perfect phase matching of all the generated components, which means that the nonlinear effect is cumulative, leading to non-negligible intensity if the mirror thickness is large enough. Phase conjugation with bubbly liquids was demonstrated using this four-wave mixing configuration [10–12].

In this article, we investigate another scheme for phase-conjugation, in which the phase matching is lost, but the second order nonlinear response of the material is exploited. As depicted in the bottom right part of Fig. 1, the idea is to use only one pump

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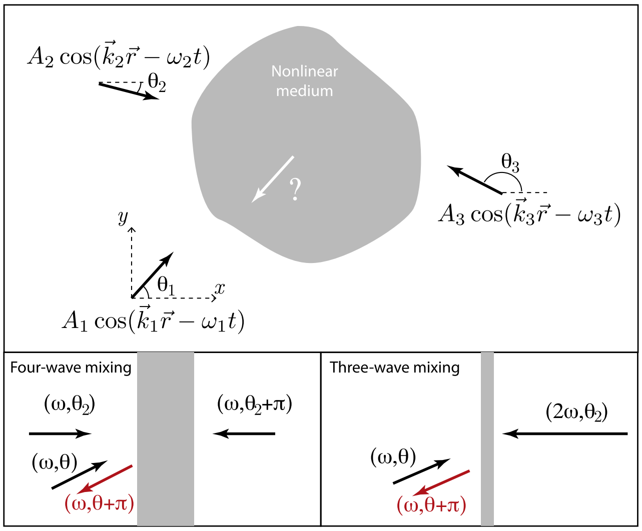

Fig. 1. Top: We consider the general case of three acoustic plane waves interacting in a nonlinear medium. Under certain circumstances, the nonlinear interaction results in a phase conjugated (PC) wave which counter-propagates with respect to one of the waves (wave 1, for instance, as depicted by the white arrow here). Bottom: two different schemes to generate a PC-wave. On the left, the four-wave mixing uses two pump waves at frequency ω and with opposite wavevectors that interact with an incoming probe wave, at the same frequency, to generate a PC-wave. On the right, the three-wave mixing scheme investigated in this article, in which there is only one pump wave at frequency 2ω.

wave at frequency $\omega_2 = 2\omega_1$, and at normal incidence (x direction, $\theta_2 = 0$). The nonlinear phase field of interest is then

$$\begin{aligned}\varphi_{+-} &= (\vec{k}_1 - \vec{k}_2)\cdot\vec{r} - (\omega_1 - \omega_2)t,\\ &= -k_2 x + \vec{k}_1\cdot\vec{r} + \omega_1 t.\end{aligned} \tag{4}$$

As in Eq. (3), we obtain a PC wave (with a $+\omega_1 t$ time dependence), but now there is an extra phase term $-k_2 x$ which means that each layer of the mirror will contribute with a different phase, making a constructive effect impossible. As a consequence, within this three-wave mixing scheme, the mirror thickness needs to be sub-wavelength. A single layer of bubbles seems a good candidate, as bubbles are known to be efficient sub-wavelength nonlinear acoustic sources.

The aim of this article is to demonstrate that a single layer of bubbles can indeed behave as a PC-mirror. We also show that our measurements are well predicted by the perturbative nonlinear second-order response that we proposed in a previous article [13].

2. Experimental setup

As a PC-mirror, we used the same system as in our previous experiments [13]: a single layer of bubbles entrapped in a yield-stress fluid placed in a thin-wall cell (Fig. 2b). The yield-stress of the fluid had to be high enough to ensure the trapping of the bubbles, and the viscosity as low as possible to limit damping of the bubble resonance. In practice, we used a commercial hair gel, diluted in two volumes of water. For $a = 70$ μm-radius bubbles separated by a distance of $d = 4$ mm, the nonlinear response of the layer of bubbles was found to be maximal at 40 kHz (i.e. at the resonance frequency of the bubbles [14]). The experimental setup was thus designed with a probe wave at 40 kHz and a pump wave at 80 kHz.

Fig. 2a shows the experimental setup. In a large tank of water, the layer of bubbles was insonified by a 40 kHz probe wave and a 80 kHz pump wave. The layer of bubbles was mounted on an absorbing screen with a 5 cm aperture, to limit edge effects due to the finite-size layer of bubbles. The reflected signal was measured by connecting the probe wave transducer to an oscilloscope (with a clipper to avoid saturation). A calibrated hydrophone (B&K 8103, 27.6 μV/Pa) was also used to determine the amplitudes of the generated pressure fields at position $\vec{r} = \vec{0}$ (*i.e.* where the center of the mirror was placed, see Fig. 2a).

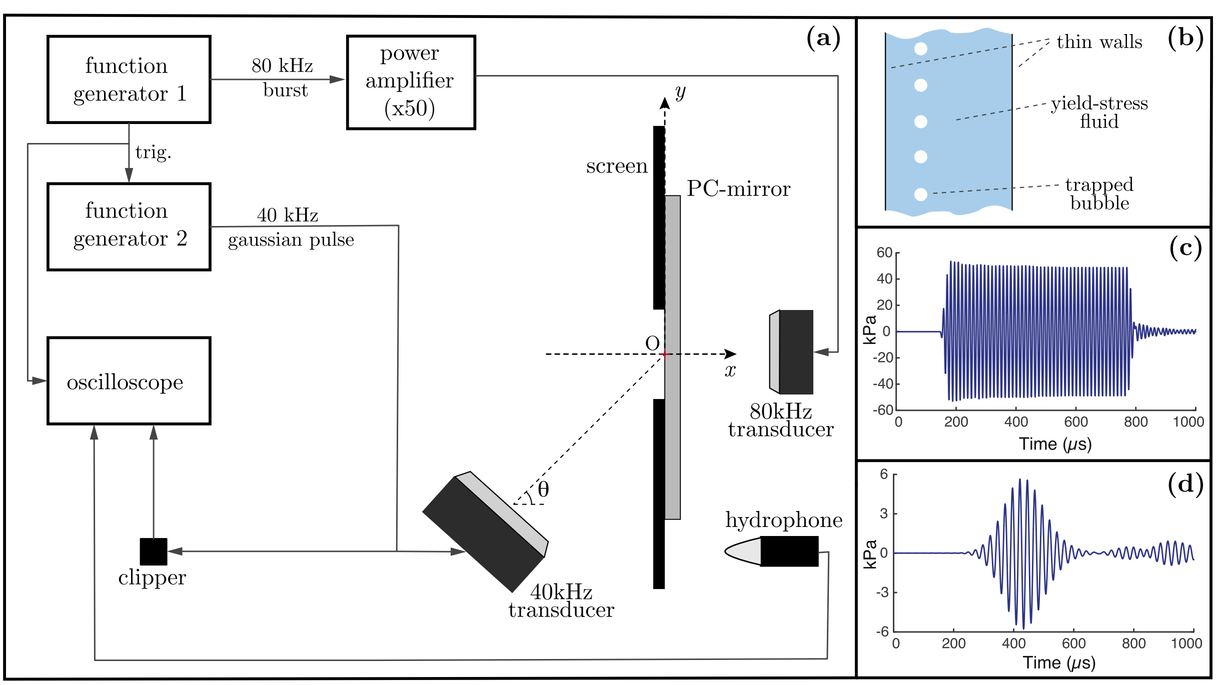

Fig. 2. Experimental setup. (a) A 40 kHz transducer generates the probe wave, while a 80 kHz one is used for the high amplitude pump wave. Signals that are reflected back to the 40 kHz transducer can be acquired by an oscilloscope, and a hydrophone is also used to scan the wave field. The center of the mirror (O) is chosen as the origin of coordinates $\vec{r} = \vec{0}$. (b) The nonlinear acoustic medium used as a PC-mirror consists of a single layer of bubbles entrapped in a yield-stress fluid. (c) Example of a pump signal, here with a 50 kPa amplitude, measured by the hydrophone at $\vec{r} = \vec{0}$ position, in the absence of the mirror. (d) Probe wave, also measured at $\vec{r} = \vec{0}$ by the hydrophone.