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Ultrasonic field modeling in anisotropic materials by distributed point source method

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ABSTRACT

DPSM (distributed point source method) is a modeling technique which is based on the concept of Green's function. First, a collection of source and target points are distributed over the solution domain based on the problem description and solution requirements. Then, the effects from all source points are superimposed at the location of every individual target point. Therefore, a successful implementation of DPSM entails an effective evaluation of Green's function between many pairs of source and target points. For homogeneous and isotropic media, the Green's function is available as a closed-form analytical expression. But for anisotropic solids, the evaluation of Green's function is more complicated and needs to be done numerically. Nevertheless, important applications such as defect detection in composite materials require anisotropic analysis. In this paper, the DPSM is used for ultrasonic field modeling in anisotropic materials. Considering the prohibitive computational cost of evaluating Green's function numerically for a large number of points, a technique called "windowing" is suggested which employs the repetitive pattern of points in DPSM in order to considerably reduce the number of evaluations of Green's function. In addition, different resolutions of numerical integration are used for computing Green's function corresponding to different distances in order to achieve a good balance between time and accuracy. The developed anisotropic DPSM model equipped with windowing technique and multi-resolution numerical integration is then applied to the problem of ultrasonic wave modeling in a plate immersed in a fluid. The transducers are placed in the fluid on both sides of the plate. First an isotropic plate is considered for the sake of verification and rough calibration of numerical integration. Then a composite plate is considered to demonstrate applicability and effectiveness of the developed model for simulating ultrasonic wave propagation in anisotropic media.

1. Introduction

Ultrasonic acoustic waves can be used for non-destructive evaluation (NDE) of materials. Sensors transmit and receive the acoustic waves through a coupling medium. The received signal is then analyzed to detect imperfections in materials. Numerical simulation of this process is of importance for NDE applications. Distributed Point Source Method (DPSM) is an effective modeling technique for solving ultrasonic wave problems [8,7].

For ultrasonic NDE of a solid plate, it is a common practice to immerse the plate in a fluid. The ultrasonic waves are transferred from the transducer to the fluid and then from the fluid to the solid. The details of DPSM modeling for this type of problem as well as many other types of problem have been given by Placko and Kundu [8]. DPSM has been used for modeling this configuration for a homogenous isotropic solid plate having a planar interface with the fluid. A finite-size scatterer can be added to this problem by placing a collection of point sources on its surface. This can represent the problem of scattering by an internal anomaly in the plate [8]. In [3] a circular cylindrical hole was added to a solid half-space, and in [10] an elliptical cavity was considered. The interface between fluid and solid can have an arbitrary shape in principle. Non-planar and relatively complex geometries for solid-fluid interface can be defined in this way [8,4,2]. In all of these studies the solid is assumed to be homogeneous and isotropic for which the solution building blocks (Green's functions) are readily available as a closed form analytical expression. The purpose of the present study is to apply DPSM for modeling ultrasonic waves in an anisotropic material.

In DPSM, the transducers and the solid-fluid interfaces are represented by a collection of point sources. Then, a global system of equations is constructed by formulating the interactions between

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these point sources, and by imposing the constraint conditions. The solution to this system of equations gives the source strengths of all point sources. These source strengths can then be used to superimpose the effect of all point sources on an arbitrary target point, thus giving the solution anywhere within the solution domain. During this process, the Green's function serves as the building block to compute the effect of one point on another point. DPSM commonly concerns with time-harmonic steady-state solutions, and thus requires elastodynamic time-harmonic Green's function as its building block.

For an anisotropic material, the elastodynamic Green's function should be evaluated numerically. Wang and Achenbach [11,12] formulated elastodynamic time-harmonic Green's functions for anisotropic solids using Radon transform. They applied Radon transform to space variables in order to convert the governing equations from a system of coupled PDEs to a system of coupled ODEs. Then, the coupled ODEs were uncoupled by transforming the coordinates to a new set of bases. Next, the uncoupled ODEs were solved, and the solution was transformed back to the original coordinate system. Then, the inverse Radon transform was applied to obtain the Green's function. The result consists of two integrals. One of them contains the singular term and the other one has only the non-singular or regular term. The singular term is in the form of an integral over an inclined circle on a unit sphere. The regular term is in the form of an integral over the surface of a hemi-sphere, and is responsible for the majority of the computational time for evaluating the anisotropic Green's function.

The singular term is similar in form to elastostatic Green's function. It can be reduced to a summation of algebraic terms using the calculus of residue. This method was used by Dederichs and Liebfried [5] and was later revived by Sales and Gray [9] who successfully used this method for calculating the Green's function as well as its first and second derivatives. This method assumes that the roots are distinct and needs special attention when repeated roots occur for a point. In the case of repeated roots, Sales and Gray [9] suggested to perturb the point by a small amount in different directions, and then use the average as an approximation.

In this paper, the solution method developed by Wang and Achenbach [12] for elastodynamic time-harmonic Green's function in anisotropic media is adopted. The integral representing the singular term is evaluated analytically using the calculus of residue based on the work of Sales and Gray [9]. The integral representing the regular term is computed numerically. As mentioned before, the regular term is responsible for the majority of the computational cost associated with the evaluation of elastodynamic Green's function. For the case of a transversely isotropic material, the integration domain for the regular part of the solution can be reduced from a hemi-sphere to a quarter-sphere as was shown by Fooladi and Kundu [6]. This improvement is very effective in reducing the computational time, and is used here for computing the Green's function of a transversely isotropic material.

The Green's function solutions are then used as the building blocks in a DPSM model to simulate ultrasonic wave propagation in an anisotropic material. This application requires numerous evaluations of Green's function, and can become prohibitively time consuming when the Green's function needs to be evaluated numerically. A technique named "windowing" is suggested in this paper which considerably reduces the number of evaluations of Green's function. In addition, different resolutions of numerical integration are used for computing Green's function corresponding to different distances so that a good balance between time and accuracy is achieved. The developed anisotropic DPSM model equipped with windowing technique and multi-resolution integration is then applied to model ultrasonic waves in a solid plate immersed in fluid. The transducers are placed in fluid on both sides of the composite plate. First an isotropic plate is considered to verify the results of anisotropic DPSM code with its isotropic counterpart. Then, a composite plate is considered and the anisotropic DPSM model is applied to it. Numerical results demonstrate applicability and effectiveness of the developed model.

The remainder of this paper is organized in the following manner. The next section is devoted to DPSM modeling. In Section 2.1 the DPSM is formulated for a solid plate immersed in a fluid, and in Section 2.2 a technique called "windowing" to reduce the computational time is presented. Next, Section 3 is devoted to elastodynamic Green's function. In Section 3.1, the general formulation for an anisotropic material is presented, in Section 3.2 the use of residue method for singular part of the solution is discussed, and in Section 3.3 the reduction of integration domain to quarter-sphere for transversely isotropic material is addressed. After presenting the formulations in Sections 2 and 3, the numerical results are shown in Section 4 where isotropic and transversely isotropic materials are considered in Sections 4.1 and 4.2, respectively. Lastly, in Section 5 the conclusion is given.

2. Distributed point source method (DPSM)

2.1. A solid plate immersed in fluid

In this subsection, the formulation of DPSM for a solid plate immersed in a fluid will be briefly described. The details of such formulation are given in Placko and Kundu [8] and Banerjee and Kundu [1].

As shown in Fig. 1, a solid plate is in contact with fluid 1 on the bottom surface, and fluid 2 on the top surface. Two flat square transducers are placed below and above the solid plate, in fluid 1 and fluid 2. A collection of source points are distributed in both fluid and solid, at the locations of transducer surface and fluid-solid interface. In Fig. 1, only the point sources along a vertical plane are shown for the sake of clarity. Several sets of point sources are identified in this figure. A source strength vector is assigned to each set of point sources, as listed in Table 1.

Each point source located in fluid has a scalar value of source strength corresponding to the pressure at that point. Each point source located in solid has three values of source strength corre-

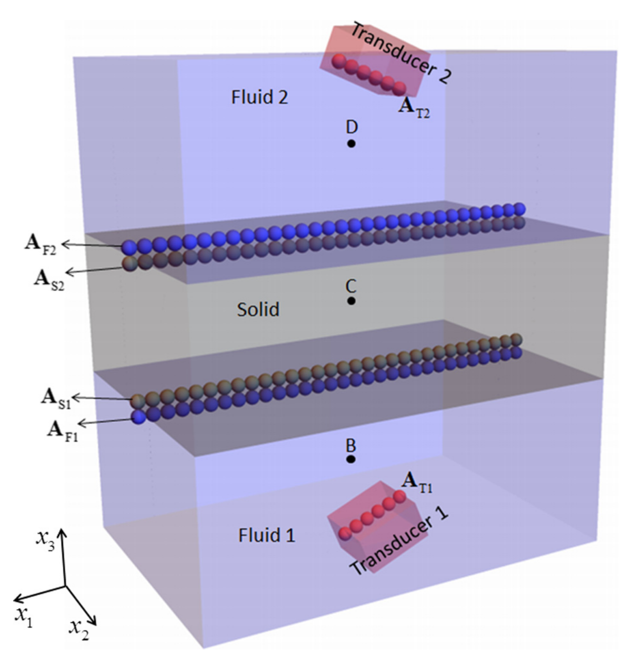

Fig. 1. Problem description and distribution of point sources.