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Generation mechanism of nonlinear ultrasonic Lamb waves in thin plates with randomly distributed micro-cracks

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Since the identification of micro-cracks in engineering materials is very valuable in understanding the initial and slight changes in mechanical properties of materials under complex working environments, numerical simulations on the propagation of the low frequency S_0 Lamb wave in thin plates with randomly distributed micro-cracks were performed to study the behavior of nonlinear Lamb waves. The results showed that while the influence of the randomly distributed micro-cracks on the phase velocity of the low frequency S_0 fundamental waves could be neglected, significant ultrasonic nonlinear effects caused by the randomly distributed micro-cracks was discovered, which mainly presented as a second harmonic generation. By using a Monte Carlo simulation method, we found that the acoustic nonlinear parameter increased linearly with the micro-crack density and the size of micro-crack zone, and it was also related to the excitation frequency and friction coefficient of the micro-crack surfaces. In addition, it was found that the nonlinear effect of waves reflected by the micro-cracks was more noticeable than that of the transmitted waves. This study theoretically reveals that the low frequency S_0 mode of Lamb waves can be used as the fundamental waves to quantitatively identify micro-cracks in thin plates.

1. Introduction

Material property degradation and defects often occur in engineering structures due to fatigue load. Thus it is necessary to develop non-destructive testing methods to evaluate the safety of engineering structures. Especially in the aerospace industry, to ensure the safety and durability of engineering structures, there is an increasing demand for the detection of micro-cracks at an early stage.

The linear ultrasonic testing technology has been widely used to detect cracks, holes, corrosion and other defects in materials, but it is only sensitive to severe defects [1–3]. When the defect is relatively small, it is difficult for conventional linear ultrasonic inspection methods to obtain the linear variation characteristics. For example, if there are closed micro-cracks in materials, the reflection and scattering at the defect cannot be inspected when the ultrasonic waves pass through them. Therefore, it is highly possible that the linear ultrasonic testing technology would fail to detect the closed micro-cracks [4,5]. Compared with the linear ultrasonic testing technology, the nonlinear characteristics of the ultrasonic waves are of high sensitivity to the microstructure of the materials with a size much smaller than the wavelength of the ultrasonic waves. The change of the microstructure due to early material degradation, such as micro-cracks, dislocations and precipitates, can be measured by nonlinear ultrasonic testing technologies [6–11]. Thus, the early detection of material property degradation and micro-cracks can be realized [12–16]. The key to the nonlinear ultrasonic testing technology for damage inspection is the excitation and detection of the nonlinear effects of the ultrasonic waves. Based on different nonlinear effects, many different nonlinear ultrasonic techniques have been developed, including higher harmonics technique [17], sub-harmonic and DC response technique [18], nonlinear resonant ultrasound spectroscopy technique, and mixing resonant wave technique [8,19]. Many studies have demonstrated that the nonlinear ultrasonic testing technology is an effective method to evaluate early material degradation and micro-crack initiation by analyzing the nonlinear effects of ultrasonic waves [20–22].

Because of the long propagation distance of the Lamb waves in a thin plate structure, nonlinear Lamb-wave detection techniques have attracted more and more attention in recent years [17,23–31]. Many efforts have been devoted to the micro-crack detection

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by higher harmonics of Lamb waves. The interactions between elastic waves and micro-cracks with clapping mechanisms have been used to explain the higher harmonic generation of Lamb waves [14,32–35]. It is believed that when elastic waves reach a micro-crack, the crack can be closed and opened under compression and tension; the compression part of the Lamb waves can penetrate the crack, while the tension part cannot. The higher harmonics are generated due to the apparent local stiffness changes at the crack region under tension and compression. However, it is challenging to detect the higher harmonics of Lamb waves due to the dispersion of Lamb waves. Deng and Lima have reported that obvious second harmonic can be observed only when phase velocities between the fundamental and double frequency Lamb waves are matched [23,24]. At present, in most studies, S_1 and A_1 modes are selected as the fundamental waves [23,24,27,36] because the condition of the phase velocity matching for these two modes can be easily satisfied. However, the research on S_0 and A_0 modes, which have higher energy, is still rarely reported [28]. Therefore, we investigated the mechanism of the second harmonic generation and propagation when S_0 mode was used as the fundamental waves. The generation condition of the second harmonic for S_0 mode was investigated by numerical modelling of S_0 mode propagation in a thin plate structure with distributed micro-cracks, and the quantitative relationship between the acoustic nonlinear parameter and the characteristics of micro-cracks was also studied. This work will provide a theoretical basis for nonlinear Lamb-wave detection techniques based on S_0 mode.

2. Linear Lamb waves

Lamb waves are most commonly used in acoustic guide-wave modes for thin plate structures, whose wavelength has the same magnitude as the thickness of the plate. The dispersion is the significant characteristic of Lamb waves, that is, the velocity of propagation depends on not only the elastic constants and density of the material, but also the frequency of waves. Fig. 1 shows the dispersion curves of Lamb wave propagation in a 2 mm aluminum plate.

Lamb waves are always composed of various wave modes, which include at least two modes (S_0 and A_0 modes) in the low-frequency domain (0–800 kHz). It is known that the zero-order modes (S_0 and A_0 modes) carry more energy than the higher-order modes, with a smaller energy attenuation during propagation compared to the higher-order modes. Moreover, Fig. 1(a) shows that the phase velocity of S_0 mode basically slowly changes in the low-frequency domain (0–800 kHz) and high-frequency domain (1800–4000 kHz). Many studies have pointed out that one necessary condition to obtain obvious second harmonic is the same of phase velocities between the fundamental and double frequency Lamb waves. Therefore, S_0 mode has a great advantage in evaluating early material degradation and micro-crack initiation in materials. Here we investigated the potential of S_0 mode within the range of 800 kHz to detect the micro-cracks in thin plate structures.

3. Acoustic nonlinearity parameter

A large number of experiments have demonstrated that the nonlinear ultrasonic detection technique based on second harmonics is an effective tool to detect the change of the microstructure of the metal materials [37]. The second harmonic waves can be generated when the fundamental ultrasonic waves pass through a material with changed microstructure. The amplitude of the second harmonic waves can be quantitatively evaluated by acoustic nonlinearity parameter.

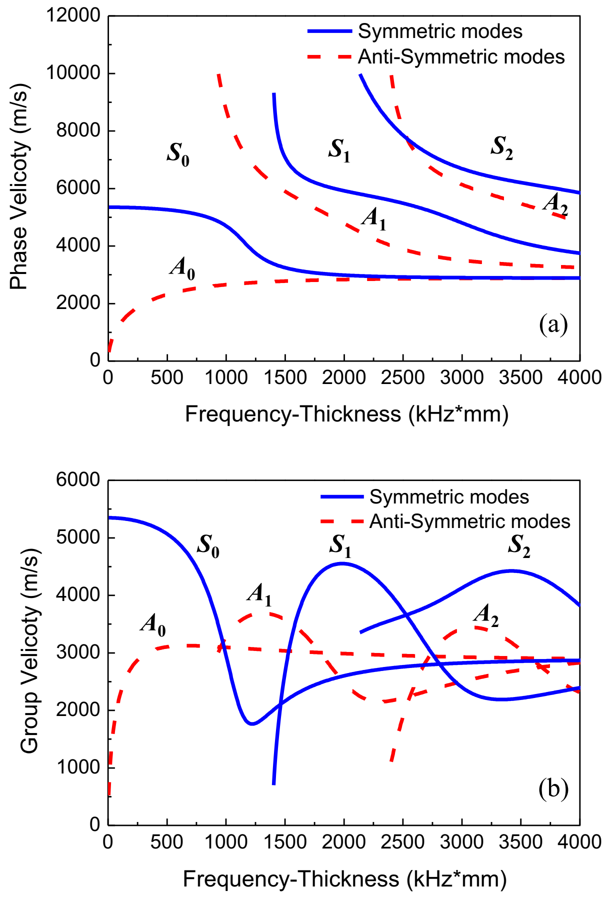

Fig. 1. Dispersion curves of Lamb waves of a 2 mm aluminum plate: (a) phase velocity; (b) group velocity.

For one dimensional case, the nonlinear stress-strain relationship based on the classical nonlinear elastic theory can be expressed as:

$$\sigma_{xx} = E_0 \varepsilon_{xx}(1 + \beta \varepsilon_{xx}) \tag{1}$$

where σ_{xx} is the stress, E_0 is the linear elastic modulus of materials, ε_{xx} is the strain, and β is the second-order classical nonlinearity parameter.

Combining the wave motion equation with Eq. (1), one dimensional nonlinear wave motion equation of longitudinal waves can be derived. Thus, the relationship between the nonlinear response (the amplitude of harmonic waves) and the classical nonlinearity parameter can be obtained. The second-order classical nonlinearity parameter can be expressed as:

$$\beta = \frac{8A_2}{A_1^2 x \kappa^2} \tag{2}$$

where A_1 is the amplitude of fundamental waves, and A_2 is the amplitude of second harmonic waves, $\kappa = \omega / c_L$ is the wave number of fundamental waves, $\omega = 2\pi f$ is the angular frequency of fundamental waves, c_L is the velocity of longitudinal waves in materials.

However, contact acoustic nonlinearity caused by cracks is different from the classical acoustic nonlinearity. A variety of theories and models have been proposed to explain the contact acoustic nonlinearity, including the hysteretic model and bi-linear stiffness model. Qu et al. [14,33,34] improved the bi-linear stiffness model by considering different effective tensile and compressive moduli