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Assessment of accumulated damage in circular tubes using nonlinear circumferential guided wave approach: A feasibility study

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ABSTRACT

The feasibility of using the nonlinear effect of primary Circumferential Guided Wave (CGW) propagation for assessing accumulated damage in circular tubes has been investigated. For a given circular tube, an appropriate mode pair of fundamental and double frequency CGWs is chosen to enable that the second harmonic of the primary wave mode can accumulate along the circumferential direction. After the given circular tube is subjected to compression-compression repeated loading for different numbers of loading cycles, the corresponding ultrasonic measurements are conducted. It is found that there is a direct correlation between the acoustic nonlinearity parameter measured with CGWs propagating through one full circumference and the level of accumulated damage in the circular tube. The experimental result obtained validates the feasibility for quantitative assessment of the accumulated damage in circular tubes using the effect of second-harmonic generation by CGW propagation.

1. Introduction

Ultrasonic guided wave techniques provide an effective means for inspecting and monitoring various structural components [1], but generally only utilize the linear behavior of wave motion, which is ineffective in evaluating material degradation or monitoring damage prior to microcrack initiation [2]. In contrast, nonlinear ultrasonic techniques can be used to quantitatively assess material degradation or damage prior to the formation of microcracks [3–6]. In the recent two decades, for accurately assessing the changing aspects of material microstructures (damage/degradation) in structural components, the investigations of the nonlinear ultrasonic guided waves have attracted more and more attentions [7]. Deng first conducted the investigation of second-harmonic generation (SHG) of primary (fundamental frequency) guided wave propagation in solid plates, and found that the second harmonic generated might grow linearly with propagation distance [8,9]. Subsequently, many researchers conducted a series of theoretical and experimental investigations on the nonlinear ultrasonic guided waves in plates, which further revealed the physical mechanism of SHG of primary guided wave propagation and meanwhile presented some effective techniques for measuring the second harmonic generated [10–22]. Based on these investigations, some experiments conducted [2,23–27] clearly demonstrated that the level of accumulated damage in plate-like structures could be quantitatively assessed using the acoustic nonlinearity parameter β as measured with the second harmonic generated by primary guided wave propagation. In addition, the nonlinear guided waves propagating axially in cylinder-like structures (including torsional, longitudinal and flexural wave modes) have also been theoretically, numerically and experimentally investigated [28–36].

The previous investigations of the nonlinear effect of ultrasonic guided wave propagation and its applications mainly focus on the cases where ultrasonic guided waves propagate in plates [7–27], or propagate axially in cylinders [28–36]. However, as a kind of elementary guided wave mode that propagates along the circumference of the circular tubes [referred to as circumferential guided wave (CGW) mode] [37,38], few investigation has been conducted on its nonlinear effect. Recently, Gao et al. established a model to analyze the SHG effect of primary CGWs [39], and meanwhile performed an experimental observation in which the second harmonic generated did accumulate along the circumferential direction [40]. Considering the fact that the SHG effect of primary guided waves has been successfully used to evaluate damage in plate-like structures [2,23–27], it is necessary to conduct further investigation to confirm whether the nonlinear CGWs can be used to quantitatively assess damage in circular tubes. For this purpose, this study reports

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an investigation focusing on the feasibility of using the nonlinear effect of primary CGW propagation for assessing accumulated damage in circular tubes.

2. Theoretical fundamentals

The Lagrangian cylindrical coordinates (r, θ) are established for a circular tube in Fig. 1, where the tube material is assumed to be isotropic, homogeneous and dispersionless. Through the equations of mechanical boundary conditions, the dispersion relations for CGW propagation, as well as the corresponding displacement fields, can readily be determined [37–39]. The displacement field $\boldsymbol{U}^{(1)}$ of the primary CGW mode (with the driving frequency f), propagating anticlockwise along the circumference of the circular tube (see Fig. 1), can be expressed formally as [37–39]

$$\boldsymbol{U}^{(1)} = \boldsymbol{U}^{(1)}(r)\exp[\mathrm{j}n^{(1)}\theta - \mathrm{j}\omega t], \tag{1}$$

where ω is the angular frequency given by $\omega = 2\pi f$, and $n^{(1)}$ is the corresponding dimensionless angular wave number of the primary CGW mode.

When the primary CGW mode travels along the tube circumference shown in Fig. 1, there is the bulk driving force of double the fundamental frequency inside the circular tube because of the convective nonlinearity independent of the material properties and the inherent nonlinearity due to the nonlinear elastic properties of the solid [10–13,39]. The components of the second-order bulk driving force in the interior of the circular tube, denoted by $F_r^{(NL)}$ and $F_\theta^{(NL)}$, are given by [39]

$$F_r^{(NL)} = \frac{1}{r}\frac{\partial\left(rP_{rr}^{(NL)}\right)}{\partial r} + \frac{1}{r}\frac{\partial P_{r\theta}^{(NL)}}{\partial\theta} - \frac{P_{\theta\theta}^{(NL)}}{r}, \tag{2}$$

$$F_\theta^{(NL)} = \frac{1}{r}\frac{\partial\left(rP_{rr}^{(NL)}\right)}{\partial r} + \frac{1}{r}\frac{\partial P_{r\theta}^{(NL)}}{\partial\theta} - \frac{P_{\theta\theta}^{(NL)}}{r}, \tag{3}$$

where $P_{rr}^{(NL)}$, $P_{\theta r}^{(NL)}$, and $P_{\theta\theta}^{(NL)}$ are the nonlinear terms in the expression of the first Piola-Kirchhoff stress tensor, and their formal expressions in the cylindrical coordinates are, respectively, given by [39],

$$\begin{aligned} P_{rr}^{(NL)} = {} & a\left[\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} - \frac{U_\theta}{r}\right)^2 + \left(\frac{\partial U_\theta}{\partial r}\right)^2\right] + 2d\left(\frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)\frac{\partial U_r}{\partial r} \\ & + b\left(\frac{\partial U_r}{\partial r}\right)^2 + d\left(\frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)^2 + e\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} - \frac{U_\theta}{r}\right)\frac{\partial U_\theta}{\partial r} \end{aligned} \tag{4}$$

$$\begin{aligned} P_{\theta\theta}^{(NL)} = {} & a\left[\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} - \frac{U_\theta}{r}\right)^2 + \left(\frac{\partial U_\theta}{\partial r}\right)^2\right] + b\left(\frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)^2 \\ & + 2d\frac{\partial U_r}{\partial r}\left(\frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right) + d\left(\frac{\partial U_r}{\partial r}\right)^2 + e\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} - \frac{U_\theta}{r}\right)\frac{\partial U_\theta}{\partial r} \end{aligned} \tag{5}$$

and

$$\begin{aligned} P_{r\theta}^{(NL)} = {} & 2a\left[\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} - \frac{U_\theta}{r}\right)\left(\frac{\partial U_r}{\partial r} + \frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)\right] \\ & + e\left[\frac{\partial U_\theta}{\partial r}\left(\frac{\partial U_r}{\partial r} + \frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)\right]. \end{aligned} \tag{6}$$

In Eqs. (4)–(6), $U_r = \hat{\boldsymbol{r}}\cdot\boldsymbol{U}^{(1)}$ and $U_\theta = \hat{\boldsymbol{\theta}}\cdot\boldsymbol{U}^{(1)}$ are, respectively, the components of $\boldsymbol{U}^{(1)}$ along the radial and circumferential directions, where $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{\theta}}$ are, respectively, the unit radial and circumferential vectors. The parameters a, b, d, and e in Eqs. (4)–(6) are derived and listed as follows [39]:

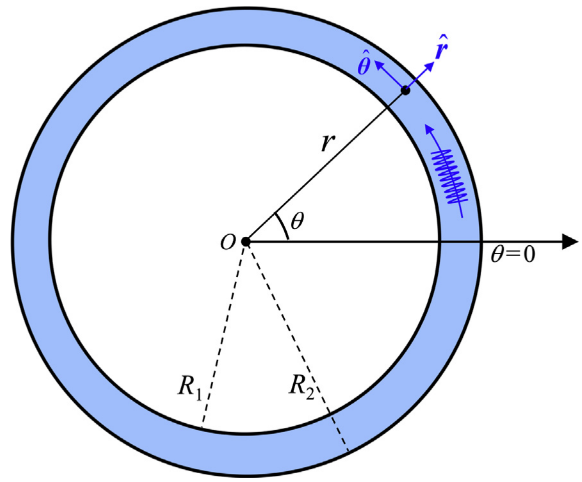

Fig. 1. Lagrangian cylindrical coordinates (r, θ) established for analyzing CGW propagation.

$$\begin{cases} a = A/4 + B/2 + \lambda/2 + \mu \\ b = A + 3B + C + 3\lambda/2 + 3\mu \\ d = B + C + \lambda/2 \\ e = A/2 + B + \mu \end{cases}, \tag{7}$$

where λ and μ are the second-order elastic constants, and A, B, and C are the third-order elastic constants defined by Landau and Lifshitz [10–12]. Obviously, there is the factor $\exp[\mathrm{j}2n^{(1)}\theta - \mathrm{j}2\omega t]$ in $F_r^{(NL)}$ and $F_\theta^{(NL)}$. Besides $F_r^{(NL)}$ and $F_\theta^{(NL)}$, there are the traction stress tensors of double the fundamental frequency, denoted by $P_{rr}^{(NL)}$ and $P_{\theta r}^{(NL)}$, on the two surfaces of the circular tube. $P_{rr}^{(NL)}$ is shown in Eq. (4) and $P_{\theta r}^{(NL)}$ is given by [39],

$$\begin{aligned} P_{\theta r}^{(NL)} = {} & 2a\left[\frac{\partial U_\theta}{\partial r}\left(\frac{\partial U_r}{\partial r} + \frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)\right] \\ & + e\left[\left(\frac{1}{r}\frac{\partial U_r}{\partial\theta} - \frac{U_\theta}{r}\right)\left(\frac{\partial U_r}{\partial r} + \frac{1}{r}\frac{\partial U_\theta}{\partial\theta} + \frac{U_r}{r}\right)\right]. \end{aligned} \tag{8}$$

Obviously, $P_{rr}^{(NL)}$ and $P_{\theta r}^{(NL)}$ also include the factor $\exp[\mathrm{j}2n^{(1)}\theta - \mathrm{j}2\omega t]$.

According to a modal analysis approach for waveguide excitation [10–13], $F_q^{(NL)}$ and $P_{qr}^{(NL)}$ ($q = r$, θ) can be assumed to be a bulk source and a surface source for generation of a series of double frequency CGW modes under a second-order perturbation approximation. The sum of these double frequency CGW modes constitutes the second-harmonic field (denoted by $\boldsymbol{U}^{(2)}$) of the primary CGW propagation, namely [39],

$$\boldsymbol{U}^{(2)} = \sum_m A_m(\theta)\times\boldsymbol{U}^{(2f,m)}(r), \tag{9}$$

where $A_m(\theta)$ is the expansion coefficient of the field function [denoted by $\boldsymbol{U}^{(2f,m)}(r)$] of the mth double frequency CGW.

On the basis of the reciprocity relation and the orthogonality of guided wave modes, the equation governing the expansion coefficient $A_m(\theta)$ can be expressed by [39]

$$\left(\frac{\partial}{\partial\theta} - \mathrm{j}n^{(2)}\right)A_m(\theta) = \frac{1}{4P_{mm}}\left[f_m^{\mathrm{surf}}(\theta) + f_m^{\mathrm{vol}}(\theta)\right], \tag{10}$$

where $n^{(2)}$ and P_{mm} are, respectively, the dimensionless angular wave number and the average power flow (per unit width, perpendicular to the tube section) of the mth double frequency CGW. Here