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A rapid method for calculating maximal and minimal inter-satellite distances

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Abstract

This paper proposes a method to rapidly and effectively calculate the maximal and minimal distances between a pair of satellites in which the leader is in an elliptic orbit. The principal idea of this method is simplifying the nonlinear squared distance function into a Taylor series with limited orders and further fitting the trigonometric functions in the derivative function of the simplified squared distance by piece-wise quadratic polynomials. By solving the zero-crossing points of the fitted quadratic curve, the critical points of the original nonlinear distance function are approximately determined. It turns out that the accuracy of the obtained solutions of the extreme distances depends on the number of intervals of the polynomial fitting. The bigger the number of intervals is, the better the accuracy. However, it is also noticed that the number of intervals is not necessary too big. For real applications a small value (e.g. 8) may be enough for the number of intervals. Besides, the method is apparently more effective for the small eccentricity cases. Finally, some simulations are further carried out to demonstrate the performances of this new method.

Keywords: Inter-satellite distance; Relative motion; Maximal distance; Minimal distance; Taylor series

1. Introduction

The maximal and minimal inter-satellite distances are the important performance parameters that are often used as the metrics representing the size of satellite formation. In real applications, the knowledge of the maximal distance is important to determine or evaluate the performance of the inter-satellite communication or relative navigation. This is because that a too large maximal distance will cause the communication interrupt or navigation failure (Gurfil and Kholshevnikov, 2006). At the same time, the knowledge of the minimal distance is important to analyze potential collision risks. In real applications of satellite formation, the relative motion between two satellites is often designed to be naturally bounded or periodical. Therefore, the resulting inter-satellite distance is also bounded or periodical. In this case, the maximal and minimal distances are in fact dependent on the initial relative states, such as the position and velocity vectors. Hence, the initial relative states of satellites definitely determine how far the follower will be from the leader from that moment on. However, the solutions of the maximal and minimal distances under given initial relative states are still absent till now. Kholshevnikov and Vassiliev (1999, 2004), Baluyev and Kholshevnikov (2005) and Kholshevnikov (2008) once studied the problem of inter-satellite distance, but they did not get the solutions of the maximal and minimal distances. Armellin et al. (2010) proposed an optimization

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Nomenclature

x	radial component of the follower's position	s	tangent of half of the true anomaly
y	along-track component of the follower's position	p_k	coefficients related to the cosine function of the Taylor series
z	cross-track component of the follower's position	q_k	coefficients related to the sine function of the Taylor series
c_{ij}	coefficients of the relative motion related to the initial relative states	F_k	symbol of the kth trigonometric functions in the Taylor series
d_{ij}	generated coefficients from c_{ij} to describe relative motion	S_j	the jth interval of the true anomaly
a	semi-major axis of the leader	M	the number of intervals
e	eccentricity of the leader	a_{kj}	second order coefficient in the fitting quadratic curve
f	true anomaly of the leader	b_{kj}	first order coefficient in the fitting quadratic curve
E	eccentric anomaly of the leader	c_{kj}	constant in the fitting quadratic curve
d_H	integration constant	f_j^*	the calculated critical point
d	inter-satellite distance between the follower and the leader	d_{max}	the maximal distance
w	squared distance	d_{min}	the minimal distance
w'	derivative of w with respect to the true anomaly		

method to numerically calculate the critical points of the distance function by the global optimizer based on Taylor models. This method is somewhat effective for this problem but it is time consuming. Our team recently proposed some methods to quantify the maximal and minimal distances (Dang et al., 2014, 2015). In Dang et al. (2014), the extreme distance problem was transformed into a root problem of 4th order polynomial. However, the effective method for solving the roots of 4th order polynomial was not further presented in Dang et al. (2014). Rather than obtaining the general solutions, the maximal and minimal distances in the coplanar case were solved analytically in Dang et al. (2015). Similarly, Wang and Nakasuka (2012) once obtained the solution of the maximal distance in a special flying-around formation.

Considering the existent problems in the current research, this paper tries to give a rapid method to look for solutions of the maximal and minimal distances in the general case of the relative motion where the leader is in an elliptic orbit. Firstly, the bounded satellite relative motion equation, i.e. the periodical solution of the Tschauner-Hempel (TH) equation (Inalhan et al., 2002), is used to model the distance of a pair of satellites. Then, the squared distance function is simplified as a second order Taylor series with respect to the eccentricity. The derivative of this simplified squared distance function (i.e. the Taylor series) is further derived. The trigonometric functions in the derivative function are fitted by quadratic polynomials. Finally, the critical points are obtained analytically by solving the zero-crossing points of the fitting quadratic curve in each interval of the independent variable. The maximal and minimal distances are found by evaluating the values of the original nonlinear distance function at all the possible critical points. To demonstrate the validity and performance of the proposed method in this paper, some numerical examples are presented and analyzed. The simulation results demonstrate that the method here is effective and accurate for rapidly calculating the maximal and minimal distances when the leader's eccentricity is small ($e < 0.1$).

2. Bounded satellite relative motion equations

The extreme distance problem in this paper is only considered in the scenario of bounded satellite relative motion. This is because the unbounded relative motion has naturally no limited distance. Hence, the equations for bounded relative motion are necessary to be introduced here firstly.

2.1. General satellite relative motion

The relative motion for a pair of satellites that both follow Keplerian orbits in a central gravitational field can be described by the following T-H equations (Inalhan et al., 2002):

$$x = \sin f\left[d_1 e + 2d_2 e^2 H(f)\right] - \cos f\left[\frac{d_2 e}{k^2} + d_3\right] \tag{1}$$

$$y = \left[d_1 + 2d_2 eH(f) + \frac{d_4}{k}\right] + \sin f\left[\frac{d_3}{k} + d_3\right] + \cos f\left[d_1 e + 2d_2 e^2 H(f)\right] \tag{2}$$

$$z = \sin f\frac{d_5}{k} + \cos f\frac{d_6}{k} \tag{3}$$

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