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External stability of a resonance during the descent of a spacecraft with a small variable asymmetry in the martian atmosphere

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Abstract

We consider the problem of a spacecraft's descent with a small asymmetry in the rarefied martian atmosphere at low values of the angle of attack. The use of the method of averaging and conditions of external stability of the main resonance make it possible for us to determine several characteristic cases of occurrence of external stability and instability during rotation of the spacecraft. It is shown, that control of the asymmetry in the above-described problem can provide stabilization of rotation of the spacecraft.

Keywords: Stability; Spacecraft; Atmosphere; Averaging; Resonance; Asymmetry

1. Introduction

It should be noted that a variety of resonance phenomena occurring in the motion of solid bodies relative to the center of mass is described in a considerable number of publications. In particular, nonlinear vibrations caused by parametric and internal resonances which occur at the loading of viscoelastic plates are studied in Tang and Chen (2012). Another paper discusses main and parametric resonances in the problem of a rotation with rapid fluctuations of an asymmetrical shaft (Shahgholy and Khadem, 2013). In addition, analysis of rotational stability of an asymmetric shaft with unequal moments of inertia and bending stiffness near the resonance is considered in Shahgholy and Khadem (2012). The resonance ratios of characteristic frequencies in the orbital motion of celestial bodies and their stability are explored in a great number of publications, for example (Elipe et al., 2012; Proshkin and Filippova, 2013; Érdi et al., 2012). However, resonances in celestial mechanics are independent problems of nonlinear dynamics, different from the problems presented in this study. The necessary and sufficient conditions for the stability of steady motion of a spacecraft with small mass asymmetry are explored in Morozov et al. (2016). In the latter case, the spacecraft is a system of coaxial bodies. The phenomenon of resonance capture of a lightweight capsule with the small mass-inertia asymmetry was analyzed using the Monte Carlo method (Zabolotnov, 2013). The problem of stability of perturbed motion of resonances relative to the center of mass of a descent spacecraft in the atmosphere was considered in a variety of publications (Yaroshevskiy, 1978; Shilov and Vasilyev, 1971; Zabolotnov, 1994; Clare, 1971) and other works. At the same time, stability of resonances was studied in an asymptotically small neighborhood of these resonances. In (Sadov, 1990, 1996) with regard to systems of general form with fast and slow variables, the concept of "external stability of a resonance" was introduced. Phenomenon of external stability of a resonance is accompanied by evolution of slow variables on non-resonant parts of the

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movement, adjacent to the considered resonance. At the same time, externally stable resonance is a manifold that "attracts" flight paths. On the contrary, externally unstable resonance is a manifold that "repels" flight paths. In a further study, a definition of external stability of a resonance was formulated and conditions for external stability of a resonance in a nonlinear system with slow and fast variables were obtained. The phenomenon of external stability of resonances was discovered in the problem of perturbed orbital motion of a rigid body with a strong magnet in the Earth's field (Lyubimov, 2010). With regard to the problem of descent of a spacecraft with a small asymmetry in the atmosphere the phenomenon of external stability of resonances was examined in the studies (Lyubimov, 2001, 2006). Those studies of external stability take no account of the evolution of parameters of a movement of a center of mass. In addition, the magnitude of displacements of the center of mass and aerodynamic shape asymmetry were assumed to be invariant, which slightly reduces the practical significance of the results obtained in these studies.

2. Problem statement

We consider a problem of external stability of the main resonance in the atmosphere during the descent of an asymmetric spacecraft at low angles of attack. The spacecraft is considered as a solid body with a mass of 70 kg and having a nearly conic shape. The spacecraft is subjected to two aerodynamic moments: stabilizing and disturbing. The stabilizing moment is proportional to the distance between the center of mass and the center of application of aerodynamic forces. Mechanical moments caused by small wind and mass asymmetries act as disturbances affecting the spacecraft. In studying external stability, it is necessary to consider only the main resonance. It is known that non-resonant evolution of the angular velocity at the simplest main resonance is greater than the corresponding values of evolution that take place at the resonances of higher orders. We intend to produce an analysis of external stability of the main resonance in a quasi-linear case (for small values of the angle of attack). Quasi-linear formulation of the problem makes it possible to perform a detailed approximate analysis of the condition of the external stability and to identify possible cases of stability and instability of the main resonance, which are supported by numerical results. Further, in order to stabilize the rotational motion of a spacecraft, we introduce an analytical dependence, that makes it possible to reduce the aerodynamic asymmetry in descent of a spacecraft in the atmosphere. We state the numerical results that reflect the phenomenon of external stability under constant and changing parameters of asymmetry.

3. Mathematical models

Mutual position of the coordinate system $OXYZ$ associated with the spacecraft and the orbital coordinate system $OX_KY_KZ_K$ is determined by the following three angles of orientation: the spatial angle of attack α_p, the aerodynamic roll angle φ_p and the high-speed roll angle γ_a. In the following, we omit the subscripts in these orientation angles.

The approximate quasi-linear system of equations of motion of a spacecraft with a small asymmetry of the center of mass in the atmosphere obtained in Zabolotnov (1993) is as follows:

$$\frac{da_1}{dt} = -\frac{\omega}{2\omega_a^2}a_1\frac{d\omega}{dt} - \varepsilon\frac{m^A}{2\omega_a}\cos(\theta+\theta_1), \quad (1)$$

$$\bar{I}_x\frac{d\omega_x}{dt} = -\varepsilon a_1 m_x^A\sin(\theta+\theta_2), \quad (2)$$

$$\frac{d\theta}{dt} = \omega_x - \omega_1 + \varepsilon\frac{m^A}{2\omega_a a_1}\sin(\theta+\theta_1), \quad (3)$$

$$\frac{d\omega}{dt} = \varepsilon\frac{\omega}{2q}\frac{dq}{dt}, \quad (4)$$

where ε is a small parameter characterizing the smallness of the parameters of mass and aerodynamic asymmetry, as well as the slowness of ω changing; a_1 is the amplitude of the angle of attack, which is the angle between the axis OX and the vector of angular momentum of the spacecraft; ω_x is the angular velocity of the spacecraft relative to the axis OX; $\theta = \varphi - \pi/2$; m_x^A is the parameter, that characterize the magnitude of the mass asymmetry; m^A is the parameter, that characterize the magnitude of the mass and aerodynamic asymmetries; θ_1 is the parameter, that defined the relative position of the mass-aerodynamic asymmetry and the coordinate system $OXYZ$; θ_2 is the parameter, that defined the relative position of the mass asymmetry and the coordinate system $OXYZ$;

$$m^A = \sqrt{(m_1^A)^2 + (m_2^A)^2},\ m_1^A = -\frac{\omega^2}{m_{zn}^\alpha}m_{yo}^f + \frac{\omega^2}{m_{zn}^\alpha}C_{x1}\overline{\Delta z},$$

$$m_2^A = -\frac{\omega^2}{m_{zn}^\alpha}m_{zo}^f - \frac{\omega^2}{m_{zn}^\alpha}C_{x1}\overline{\Delta y};\ \sin\theta_1 = m_1^A/m^A;$$

$$\cos\theta_1 = -m_2^A/m^A;$$

$$m_x^A = \sqrt{(m_{x1}^A)^2 + (m_{x2}^A)^2},\ m_{x1}^A = -\frac{\omega^2}{m_{zn}^\alpha}C_{y1}\overline{\Delta y},$$

$$m_{x2}^A = -\frac{\omega^2}{m_{zn}^\alpha}C_{y1}\overline{\Delta z};$$

$\sin\theta_2 = -m_{x1}^A/m_x^A$; $\cos\theta_2 = m_{x2}^A/m_x^A$; C_{x1}, C_{y1} are the coefficients of aerodynamic forces (Zabolotnov, 1993); m_{yo}^f, m_{zo}^f are the coefficients of small aerodynamic moments caused by asymmetric shape of the spacecraft; $\overline{\Delta y}, \overline{\Delta z}$ are small dimensionless shifts of the center of mass of the spacecraft in the coordinate system $OXYZ$ (Zabolotnov, 1993); ω_1 is the precession frequency; $\omega_1 = \frac{\bar{I}_x\omega_x}{2} + \omega_a$; $\bar{I}_x = I_x/I$; I_x and $I_y = I_z = I$ are the moments of inertia of the spacecraft relative the axes of the coordinate system $OXYZ$, $\omega_a = \sqrt{\frac{\bar{I}_x^2\omega_x^2}{4} + \omega^2}$; ω is the precession frequency with angular velocity equal to zero; $\omega = \sqrt{m_z^\alpha qSL/I}$; q, S, L are the