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One hybrid model combining singular spectrum analysis and LS + ARMA for polar motion prediction

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Abstract

Accurate real-time polar motion parameters play an important role in satellite navigation and positioning and spacecraft tracking. To meet the needs for real-time and high-accuracy polar motion prediction, a hybrid model that integrated singular spectrum analysis (SSA), least-squares (LS) extrapolation and an autoregressive moving average (ARMA) model was proposed. SSA was applied to separate the trend, the annual and the Chandler components from a given polar motion time series. LS extrapolation models were constructed for the separated trend, annual and Chandler components. An ARMA model was established for a synthetic sequence that contained the remaining SSA component and the residual series of LS fitting. In applying this hybrid model, multiple sets of polar motion predictions with lead times of 360 days were made based on an IERS 08 C04 series. The results showed that the proposed method could effectively predict the polar motion parameters.

Keywords: Singular spectrum analysis; Polar motion; Prediction; Least squares; Autoregressive moving average

1. Introduction

Polar motion (PM), as one of the Earth's rotation parameters, is a significant factor in the fields of space geodesy, geophysics, astronomy, and in other areas of space-related science and technology (Dobslaw et al., 2010; Štěpánek et al., 2014; Shen et al., 2015; Song and Kim, 2016). Although highly accurate PM parameters can be obtained by space geodesy techniques, the post-processed PM parameters cannot meet the real-time demands of satellite navigation and positioning and spacecraft tracking (Kalarus et al., 2010; Xu et al., 2012; Jayles et al., 2016). The performance of satellite orbital prediction is affected by errors in the PM predictions that are required for transforming from an inertial reference frame to an Earth-fixed frame (Choi et al., 2013). To meet these practical requirements, it is particularly important to predict the PM parameters precisely.

Various methods and means have been proposed for making PM predictions, such as the spectral-analysis and LS-extrapolation method (Akulenko et al., 2002), the wavelet analysis and fuzzy inference system (Akyilmaz and Kutterer, 2004; Akyilmaz et al., 2011), artificial neural networks (Schuh et al., 2002; Liao et al., 2012), LS and autoregressive (AR) filtering (Kosek et al., 1998; Kosek, 2002), excitation functions (Chin et al., 2004; Dill et al., 2013), and normal time-frequency transforms (Su et al., 2014). Most of these methods establish a harmonic model which includes trend and periodic terms, uses LS to

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extrapolate them into the future, and then forecasts a residual series by a deterministic or a stochastic model (e.g. a neural network, or AR). Among these methods, a combination of the LS extrapolation and the AR prediction models (the LS + AR model) is considered one of the most effective means of PM prediction (Kosek et al., 2007; Kalarus et al., 2010). Many scholars propose various types of combination models, based on the LS + AR model (e.g. Xu et al., 2012; Guo et al., 2013; Xu and Zhou, 2015). These models have achieved good results for short-term predictions of PM. However, due to the complexity of the PM excitation mechanism and the time-variations of the annual and the Chandler wobble (Schuh et al., 2001; Guo and Han, 2009; Zotov and Bizouard, 2015), the traditional LS extrapolation model is unable to reproduce the time variation of the periodic terms that influence the long-term predictive accuracy of the PM. Therefore, it is important to establish a suitable PM prediction model for dealing with the time variations of the periodic terms and the trend.

The SSA (Broomhead and King, 1986), as a generalized power spectrum analysis, is not constrained by the assumption of sine waves. This method of analysis can effectively identify and display the periodic signal of a time series, allowing the precise separation and reconstruction of the principal components. SSA has been used in oceanography (Kondrashov and Berloff, 2015; Jalón-Rojas et al., 2016), climatology (Wyatt et al., 2012; Rial et al., 2013), surveying (Chen et al., 2013; Zabalza et al., 2014) and in other fields that involve component analyses of time series. Additionally, the time series of the primary periods (as separated from the raw series by SSA) have the characteristics of stationary periodic variations, which make forecasting easier. The combination of SSA and other forecasting models can achieve good prediction results (Zotov, 2010; Zhang et al., 2011; Heng and Suetsugi, 2013).

Our goal was to integrate SSA and LS + ARMA for precisely forecasting the long-term PM. First, the long-term time series of PM from EOP 08 C04 was analyzed and separated by SSA. Then, a combined LS extrapolation and autoregressive moving average (ARMA) prediction model was made based on the SSA separated time series. Finally, the accuracy and reliability of the proposed hybrid method was verified through multiple sets of PM prediction tests.

2. Singular spectrum analysis of polar motion

The SSA, as a method for examining the one-dimensional nonlinear time series, constructs a trajectory matrix, and this matrix can be decomposed and reconstructed to extract various components of the original time series, such as the long term trend, the periodic terms or noise (Vautard and Ghil, 1989). In the following sections, the basic principle of SSA is introduced and the analysis of a PM time series by using SSA is presented.

2.1. Principle of SSA

(1) *Embedding*. The first step in the basic SSA algorithm is the embedding step, where the initial time series changes into the trajectory matrix. A daily time series $\{x_T\}$ has the length of T. This one-dimensional time series can be mapped into a multi-dimensional trajectory matrix with the window length L, which should be an integer that is less than $T/2$. The trajectory matrix X of this series is

$$\underset{(T-L+1)\times L}{X} = \begin{bmatrix} x_1 & x_2 & \cdots & x_L \\ x_2 & x_3 & \cdots & x_{L+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{T-L+1} & x_{T-L+2} & \cdots & x_T \end{bmatrix}. \quad (1)$$

(2) *Empirical orthogonal function (EOF) decomposition*. The covariance matrix C of X can be estimated directly from the data, whose element is

$$c_{ij} = \frac{1}{T-k}\sum_{t=1}^{T-k} x_t x_{t+k} \quad (2)$$

where $i, j = 1, 2, \ldots, L$, and $k = |i-j|$. Then the eigenvalues $\lambda_1 \geqslant \lambda_2 \geqslant \ldots \geqslant \lambda_L$ and the eigenvectors $v_1, v_2, \ldots, v_L$ of the matrix C are calculated. The EOF decomposition of the matrix X can be computed by

$$x(i,j) = \sum_{k=1}^{L} y(i,k)v(k,j) \quad (3)$$

where $i = 1, 2, \ldots, T-L+1$; $j = 1, 2, \ldots, L$; $v(k,j)$ as the set of eigenvectors of the matrix C is called the temporal empirical orthogonal function (TEOF), and $y(i,k)$ is the set of corresponding temporal principal components (TPC).

(3) *Reconstruction*. The entire time series, or the part of it that correspond to trends, oscillatory modes or noise, can be reconstructed by using linear combinations of TPCs and TEOFs. The k-th reconstructed component (RC) is

$$x_i^k = \begin{cases} \frac{1}{i}\sum_{j=1}^{i} y(i-j,k)v(k,j), & 1 \leqslant i \leqslant L-1 \\ \frac{1}{L}\sum_{j=1}^{L} y(i-j,k)v(k,j), & L \leqslant i \leqslant T-L+1 \\ \frac{1}{T-i+1}\sum_{j=i-T+L}^{L} y(i-j,k)v(k,j), & T-L+2 \leqslant i \leqslant T \end{cases} \quad (4)$$

SSA can reconstruct L components, in which the low-frequency RCs can effectively express the primary changes in the original series. The RCs should be analyzed to obtain the separated periodic RCs and the trend RC. The Kendall nonparametric test is