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New fundamental parameters for attitude representation

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Abstract

A new attitude parameter set is developed to clarify the geometry of combining finite rotations in a rotational sequence and in combining infinitesimal angular increments generated by angular rate. The resulting parameter set of six Pivot Parameters represents a rotation as a great circle arc on a unit sphere that can be located at any clocking location in the rotation plane. Two rotations are combined by linking their arcs at either of the two intersection points of the respective rotation planes. In a similar fashion, linking rotational increments produced by angular rate is used to derive the associated kinematical equations, which are linear and have no singularities. Included in this paper is the derivation of twelve Pivot Parameter elements that represent all twelve Euler Angle sequences, which enables efficient conversions between Pivot Parameters and any Euler Angle sequence. Applications of this new parameter set include the derivation of quaternions and the quaternion composition rule, as well as, the derivation of the analytical solution to time dependent coning motion. The relationships between Pivot Parameters and traditional parameter sets are included in this work. Pivot Parameters are well suited for a variety of aerospace applications due to their effective composition rule, singularity free kinematic equations, efficient conversion to and from Euler Angle sequences and clarity of their geometrical foundation.

Keywords: Attitude parameters; Attitude kinematics; Rotational transformation; Coning motion; Quaternions; Euler angles

1. Introduction

The attitude of a space vehicle can be defined by the orientation of its body fixed reference frame with respect to an inertial reference frame. Attitude parameters, which define the orientation of the body fixed reference frame, should enable a sequence of rotations to be combined easily into a single rotation, since Euler's Theorem states that any orientation can be represented by a single rotation about an Euler Axis. Attitude parameters should also permit the attitude propagation of the body when given the angular rate of the body as a function of time. The kinematical algorithm should be linear and free of singularities. In addition, it is beneficial for the parameters to have a physical or geometrical interpretation to promote conceptual understanding. Euler Angle sequences provide a geometrical representation of an attitude transformation but are cumbersome in combining a sequence of rotations and are not well suited for attitude propagation due to the use of trigonometric functions and the need to avoid the singularity that appears in all attitude parameter sets that involve the minimum required three parameters (Shuster, 1993; Schaub and Junkins, 2003). Quaternion parameters are effective in combining a sequence of rotations and in attitude propagation, but lack a conceptual foundation and geometrical interpretation that would enhance understanding. The Euler Rotation Vector, $\mathbf{\Phi}$, representation of attitude transformation is geometrically intuitive but suffers disadvantages similar to that of Euler angle sequences (Shuster, 1993; Schaub and Junkins, 2003). The Direction Cosine Matrix (**DCM**), representation effectively transforms vectors between body and inertial

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Nomenclature

a, **b**, **A**, **B**	Pivot Vectors	**U**(**PV1**,**PV2**)	direction cosine matrix for **PV** pair
a1, **b1**	Pivot Vectors	X	arbitrary vector
$\mathbf{a_C}$, $\mathbf{b_C}$	Pivot Vectors	x, y, z	coordinate axes
c, **d**	Pivot Vectors	$\boldsymbol{\lambda}$	constant slew rate of angular rate vector
DCM	direction cosine matrix	β	time derivative of α
e	unit vector	$\Delta\theta$	angular increment
f	normalization constant	γ	time derivative of coning amplitude
N	unit vector defining intersection of planes	ϕ	rotation angle
PP	Pivot Parameter	$\boldsymbol{\Phi}$	Euler vector
PV	Pivot Vector	$\boldsymbol{\Psi}$	constant angular rate vector
q	vector part of quaternion	θ	rotation angle
$q(0)$	scalar part of quaternion	θ, ϕ, α	symmetric Euler angle sequence
t	time	θ, ϕ, γ	asymmetric Euler angle sequence
$U(\mathbf{e})$	direction cosine matrix for **e**	$\boldsymbol{\omega}$	angular rate vector

reference frames, but it contains nine matrix elements that make propagation cumbersome due to the requirement to maintain matrix orthonormality (Shuster, 1993; Schaub and Junkins, 2003). Although each set of attitude representations can be effective for particular applications, no single parameter set is ideal for all applications.

In this work, attitude parameters were developed specifically to reveal the fundamental geometry of rotation and to enable a sequence of rotations to be combined easily. The resulting attitude parameters, referred to as Pivot Parameters (**PP**s), show that a rotation can be represented as a great circle arc on a unit sphere that can be located at any clocking position in the rotation plane. Two rotations can be combined by connecting their rotation arcs at the intersection of their rotation planes, which enables two sequential rotations to be combined into a single rotation very easily and by extension any sequence of rotations can be reduced to a single rotation. The algorithm for combining two rotations is derived from the fundamental property of **PP**s and geometry of the intersection of the respective great circle arcs on the unit sphere. In addition, the kinematic algorithm for propagating attitude is essentially the same as the algorithm for combining rotations, since attitude propagation involves combining infinitesimal sequential rotation increments created by the body angular rate over discrete time steps. Since the attitude propagation algorithm involves simple linear equations having no trigonometric functions or singularities, attitude propagation is accurate and efficient. These new attitude parameters are fundamentally related to DCM, Euler Vector, quaternion and Euler Angle attitude representations and shed new insight into each parameter set. Pivot Parameters were used to derive quaternions and the quaternion composite rule while providing the related geometrical foundation. All twelve Euler angle sequences were compactly represented as just twelve **PP**s. In addition, **PP**s not only reveal the geometry of coning motion, but enable the derivation of the analytical solution to coning motion with time varying amplitude and frequency.

In Section 2, a detailed overview of **PP** methodology is presented. In Section 3, the **PP** method is illustrated for rotation about a fixed axis to highlight the conceptual simplicity of the method. In Section 4, the **PP**s are developed for several important Euler angle sequences and the formulation enables a compact representation for all 12 possible Euler angle sequences. In Section 5, the **PP**s associated with the combination of two arbitrary rotations are derived. The derivation reveals how quaternions arise and how two quaternions can be combined into a single quaternion. In Section 6, the methodology is applied to angular rates by finding **PP**s associated with a body rotating with both a fixed and time vary angular rate. The resulting kinematical equations are efficient and do not have singularities or involve trigonometric functions. In Section 7, PPs reveal the geometry of coning motion and enable the derivation of the analytical solution to coning motion with time varying frequency and amplitude. Section 8 contains numerical results that demonstrate attitude propagation accuracy while validating the time dependent coning motion solution. Section 9 is the conclusion.

2. Pivot vector methodology

Pivot Parameters are not based on an angle of rotation about an axis of rotation, as are traditional parameters, such as, the Euler rotation angle about the Euler Axis, Euler Angles or the vector part of the quaternion, **q**. Pivot Parameter representation of attitude replaces a single rotation of θ about the Euler Axis with two sequential rotations of 180 degrees about axes located in the rotation plane separated by $\theta/2$ degrees and passing through the origin of the body fixed reference frame. Each of these axes is aligned with a Pivot Vector (**PV**), which extends from the origin of the body fixed reference frame to the surface of a unit

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