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Advances in Space Research xxx (2016) xxx–xxx

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Possible signature of solar oblateness in the Sun's oscillation frequency splittings

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Received 30 October 2015; received in revised form 4 March 2016; accepted 9 March 2016

Abstract

Departures from spherical symmetry split the frequencies of the Sun's normal oscillation modes. In addition to the well-studied, dominant splitting of the mode frequencies, due to the first-order advection of internal wave motion, a number of second-order effects of rotation on the frequency splittings, predominantly the solar oblateness, are expected. Whereas the largest rotational frequency splittings have an odd dependence on the azimuthal order, m, of the modes, the second-order effects should have an even dependence. The biggest, and thus far the only well-studied, even-m effect on splittings, is due to the solar-cycle variations in magnetic activity near the Sun's surface, which need to be modeled with some care to bring out the signature of solar oblateness. A crude analysis of the even mode-frequency splittings, obtained from approximately 15 years of SOHO/MDI spherical-harmonic time series, was undertaken. To extract the small even-m splittings of interest from the dominant, solar-cycle effects, which have a strong mode-frequency dependence, the former were assumed to depend only weakly on mode frequency and to have no time dependence. Perhaps the most important finding of the study is that the MDI data are capable of yielding statistically significant estimates of solar oblateness. Indeed the oblateness estimates obtained from the analysis presented here appear to be roughly consistent with both theoretical expectations and with direct measurements of the oblateness. There is also a hint of a pole-equator temperature difference in the seismic measurements, at the level recently suggested by Miesch and Hindman.

Keywords: Solar interior; Mass flows; Helioseismology; Solar dynamo

1. Background

The m dependence (splitting) of the frequencies ω_{nlm} of the Sun's normal oscillation modes reflects the nature of the solar internal angular velocity (differential rotation) and other departures from spherical symmetry of the interior state of the Sun. The mode frequency splittings are dominated by the effect of solar rotation, but in addition have been found to contain the signature of latitude-dependent structure (Duvall et al., 1986). The splittings are often parameterized by the coefficients $a_i(n, l)$ of an expansion of the mode frequencies at fixed n and l in so-called Lavely–Ritzwoller polynomials in m/L, where $L = \sqrt{l(l+1)}$. (For sufficiently large l, these polynomials are approximately equal to $LP_i(m/L)$, where P_i is the Legendre polynomial of degree i.) Whereas the dominant, first-order, mode frequency perturbations by solar rotation affect the odd-i a-coefficients, the much smaller perturbations of interest here affect the even-i coefficients, sometimes referred to as asphericity coefficients.

Kuhn (1988) noticed a connection between the seismic asphericity measurements and the latitude- and time-dependent effective temperature measurements seen near the solar limb and suggested that the seismic asphericity was produced by latitudinal bands of elevated temperature coinciding with magnetically active belts.

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http://dx.doi.org/10.1016/j.asr.2016.03.019

Please cite this article in press as: Woodard, M.F. Possible signature of solar oblateness in the Sun's oscillation frequency splittings. Adv. Space Res. (2016), http://dx.doi.org/10.1016/j.asr.2016.03.019

Subsequently, Gough and others (Gough et al., 1988; Goldreich et al., 1991; Dziembowski et al., 2000) argued that fibril magnetic fields in and just below the photosphere play an important role, either directly or indirectly, in perturbing the solar eigenfrequencies. The strong mode frequency dependence of the solar-cycle variations of the frequency splittings and (mode-multiplet) frequencies, $\nu(n,l)$, has been taken as evidence that the cycle-related frequency changes are produced mainly by near-surface magnetic activity (Libbrecht and Woodard, 1990). Like the multiplet frequency variations, the even splitting variations are much bigger at high ν than at low ν. The strong frequency dependence of near-surface perturbations can be understood qualitatively as a barrier penetration phenomenon, since the near-surface evanescent (non-propagating) region of low frequency modes extends deeper into the solar interior than that of high frequency modes, the low frequency modes are thus less sensitive to near-surface perturbations.

The analysis described below considers the coefficients $\alpha_i(n,l) = La_i(n,l)$ (defined, for instance, in Gough (1988)). The even-i α-coefficients have the desirable property of being comparable in magnitude to the observed solar-cycle variations of the m-independent (mode-multiplet) frequencies $\nu(n,l)$. (Time variations of order a few hundred nHz in $\nu(n,l)$ reflect the spherically symmetric component solar-cycle changes.) This article describes an attempt to estimate the amplitude of non-magnetic (or at least time-independent) aspherical structure, principally of solar oblateness, that should be present in helioseismic data, though at a much smaller level than the observed magnetic effects.

For simplicity only the α_2 coefficients were analyzed for this work, which are crude measures of pole-to-equator variations of magnetic activity and sound speed. Large-scale non-magnetically induced latitude variations in sound speed are expected as a consequence of the solar oblateness and other higher-order effects of rotation. As the angular velocity is observed to be fairly constant over the activity cycle, the time dependence of the small, higher-order effects can probably be neglected for the present. The second-order effect of rotation on the asphericity coefficients has been computed for solar models with the observed internal angular velocity (e.g., Dziembowski and Goode, 1992; Gough, 1988). The solar oblateness itself is found to be the dominant second-order effect on mode frequencies and the magnitude of its contribution to α_2 varies more or less monotonically with frequency from approximately 10 to 30 nHz over the 1–5 mHz mode frequency range, for the modes of $l < 300$ considered here. The sign of the oblateness contribution is negative. Therefore the solar oblateness is more likely to be detectable in the lowest-frequency modes observed than in higher-frequency modes, as the former are expected to be relatively insensitive to near-surface magnetic effects. Miesch and Hindman (2011) have pointed out that, in addition to the solar oblateness effects, there may be contributions to α_2 of similar magnitude from a pole-equator temperature difference.

2. Methodology

A rigorous comparison of the observed seismic asphericity and the predictions of a detailed physical model would be a valuable exercise. But as the goal of the present analysis is simply to investigate whether the Sun's oblateness affects measured oscillation frequencies at an observable level, a much simpler approach was taken. Accordingly, the 15 years of SOHO/MDI Medium-l α_2 measurements (Scherrer et al., 1995; Larson and Schou, 2015) were modeled as linear combinations of a 'large' and a 'small' component. The large component is intended to represent the relatively well-studied, time-dependent, and strongly frequency-dependent variations of the observed α_2 coefficients, which have previously been modeled in terms of near-surface magnetic activity. The small component, on the other hand, is meant to represent the not-well-studied seismic second-order effects of rotation, mainly the solar oblateness, described in the previous section. The latter effect is expected to be small in comparison with the magnetic effects, except at low mode frequency, as noted in the previous section. Accordingly, the small component is represented by a time-independent oblateness offset and is allowed a weak, but otherwise arbitrary, dependence on mode frequency. The treatment thus ignores the l dependence of the α_2 sensitivity to magnetic or structural perturbations.

At a given n and l, the large, near-surface, component of $\alpha_2(n,l)$ was expediently assumed to be proportional to a proxy for the effect of near-surface, fundamentally magnetic, perturbations on the modes, with a frequency-dependent coefficient of proportionality. As the high frequency modes are most sensitive to conditions near the surface, a default activity proxy was defined to be the observed α_2 averaged without weighting over modes in the 4–5 mHz frequency range. But since even high-frequency modes are expected to be affected (albeit slightly) by the oblateness, a better proxy of the near-surface effects should be obtainable by removing the oblateness contribution from the default proxy. However, the oblateness contribution is being treated here as a quantity to be measured not imposed. So it seemed more appropriate to perform the data analysis for a set of trial proxies differing from the default proxy by constant (i.e., time-independent) functions and to use the sensitivity of the fitted parameters to the assumed proxy to gauge the robustness of the measured oblateness offset. A weak prior constraint was imposed on the oblateness offset in that the trial proxies were allowed to differ from the default by no more than 30 nHz.

To implement the model fitting, modes of degree l between 0 and 300 were grouped into 0.5 mHz-wide frequency bins covering the range 1.0–3.5 mHz and the time dependence of an unweighted mean α_2 for each group was fitted to the model, yielding, for each frequency bin,

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