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Characterisation of the turbulent electromotive force and its magnetically-mediated quenching in a global EULAG-MHD simulation of solar convection

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Abstract

We perform a mean-field analysis of the EULAG-MHD millenium simulation of global magnetohydrodynamical convection presented in Passos and Charbonneau (2014). The turbulent electromotive force (*emf*) operating in the simulation is assumed to be linearly related to the cyclic axisymmetric mean magnetic field and its first spatial derivatives. At every grid point in the simulation's meridional plane, this assumed relationship involves 27 independent tensorial coefficients. Expanding on Racine et al. (2011), we extract these coefficients from the simulation data through a least-squares minimization procedure based on singular value decomposition. The reconstructed $\boldsymbol{\alpha}$-tensor shows good agreement with that obtained by Racine et al. (2011), who did not include derivatives of the mean-field in their fit, as well as with the $\boldsymbol{\alpha}$-tensor extracted by Augustson et al. (2015) from a distinct ASH MHD simulation. The isotropic part of the turbulent magnetic diffusivity tensor $\boldsymbol{\beta}$ is positive definite and reaches values of $5.0 \times 10^7\ \mathrm{m^2\,s^{-1}}$ in the middle of the convecting fluid layers. The spatial variations of both $\alpha_{\phi\phi}$ and $\beta_{\phi\phi}$ component are well reproduced by expressions obtained under the Second Order Correlation Approximation, with a good matching of amplitude requiring a turbulent correlation time about five times smaller than the estimated turnover time of the small-scale turbulent flow. By segmenting the simulation data into epochs of magnetic cycle minima and maxima, we also measure α- and β-quenching. We find the magnetic quenching of the α-effect to be driven primarily by a reduction of the small-scale flow's kinetic helicity, with variations of the current helicity playing a lesser role in most locations in the simulation domain. Our measurements of turbulent diffusivity quenching are restricted to the $\beta_{\phi\phi}$ component, but indicate a weaker quenching, by a factor of $\simeq$1.36, than of the α-effect, which in our simulation drops by a factor of three between the minimum and maximum phases of the magnetic cycle.

Keywords: Magnetohydrodynamics; Solar cycle; Convection; Beta-effect; Diffusivity quenching

1. Introduction

A proper understanding of the physical mechanism(s) underlying solar dynamo action and regulating the cycle's amplitude and duration are crucial components of long term prediction of space weather (also known as "space climate"), and of research on solar-terrestrial interaction in general (Weiss, 2010). We are still a long way from physically-based prediction of solar cycle characteristics, even though significant progress has been made in recent years (for a recent review see Petrovay, 2010). Part of the difficulty lies with the fact that no concensus currently exists as to the mode of operation of the solar cycle; the shearing of the solar magnetic field by differential rotation is usually considered as a key process, but what drives the regeneration of the solar dipole moment remains

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ill-understood. Some dynamo models invoke the electromotive force associated with turbulent convection, others the surface decay of active regions (the Babcock–Leighton mechanism), while others yet focus on various rotationally-influenced (magneto) hydrodynamical instabilities taking place immediately beneath the base of the solar convection zone. A survey of these different types of dynamo models can be found in Charbonneau (2010). Such models make use of geometrical and dynamical simplifications, most notably perhaps the use of the so-called kinematic approximation, in which the dynamical backreaction of the magnetic field on the inductive flows is neglected or parametrized through largely ad hoc prescriptions. Proper tuning of these ad hoc functionals and associated model parameters can in many cases lead to cyclic behavior showing reasonably solar-like variability patterns in the amplitude and duration of magnetic cycles (see, e.g., Karak and Choudhuri, 2011; Kitchatinov and Olemskoy, 2012 and references therein).

An alternate approach is made possible by global magnetohydrodynamical simulations of solar convection, which recently have succeeded in producing magnetic fields well-organized on large spatial scales and undergoing more or less regular polarity reversals (Brown et al., 2010, 2011; Ghizaru et al., 2010; Käpylä et al., 2010; Racine et al., 2011; Käpylä et al., 2012; Masada et al., 2013; Beaudoin et al., 2013; Passos and Charbonneau, 2014; Fan and Fang, 2014; Augustson et al., 2015). There are no active regions in such simulations (but do see Nelson et al., 2013; Nelson et al., 2014), and therefore no Babcock–Leighton mechanism, but the turbulent electromotive force associated with thermally-driven convection is captured in a dynamically consistent manner at spatial and temporal scales resolved by the computational grid. Evidence for the development of MHD instabilities has also been found in some of these simulations (see Lawson et al., 2015; Miesch, 2007).

The availability of such simulation data allows to bridge the gap between simplified kinematic models and MHD simulations of solar convection. More specifically, the latter can be used to measure turbulent coefficients usually specified in largely ad hoc fashion in the former. Of particular interest is the turbulent electromotive force, its associated α-effect and turbulent diffusivity, and variations of these as a function of the magnetic field strength. Such measurements can assist in the interpretation of simulation results, and may help in clarifying some puzzling differences in the characteristics of cycles generated by simulations that are generally alike and differ primarily in what one would have hoped are only computational and algorithmic detail (see, e.g., Section 3.2 Charbonneau, 2014). Moreover, mean-field models incorporating source terms and physical coefficients derived from numerical simulations can be useful in exploring long timescale behaviors that remain unaccessible to full MHD simulations, due to limitations in computing resources.

The aim of this paper is twofold. First, we document and validate a generalization of the least-squares minimization technique introduced by Racine et al. (2011) (see also Brandenburg and Sokoloff, 2002) for extracting mean-field coefficients from the output of global MHD simulations of solar convection. Second, we use this methodology to measure the level of magnetically-mediated quenching of the α-effect and turbulent diffusivity operating in the simulation. Section 2 presents a minimal overview of classical mean-field electrodynamics, focusing on aspects necessary to properly frame the analyses to follow. In Section 3.1 we describe the least-squares minimization method used to extract the $\boldsymbol{\alpha}$- and $\boldsymbol{\beta}$-tensors, and present the results of this procedure in Section 3.2 and 3.3, applied to the "millenium simulation" described in Passos and Charbonneau (2014). We also compare in Section 3.4 the isotropic part of these two tensors to reconstructions using analytical forms obtained under the second-order correlation approximation. In Sections 4 we turn to an investigation of the magnetic suppression of the α-effect and turbulent diffusivity. We close in Section 5 by summarizing our conclusions and discussing the limitation of our analyses.

2. Mean-field electrodynamics

The mathematical and physical underpinnings of mean-field electrodynamics are well-covered in many textbooks and review articles (see, e.g., Moffatt, 1978; Krause and Rädler, 1980; Ossendrijver, 2003; Brandenburg and Subramanian, 2005; Charbonneau, 2010). What follows is only a brief overview, focusing on definitions and reformulations of the $\boldsymbol{\alpha}$- and $\boldsymbol{\beta}$-tensors on which the analyses presented in this paper are based. The starting point of classical mean-field electrodynamics is the separation of the magnetic field ($\boldsymbol{B}$) and flow ($\boldsymbol{u}$) into a spatially large-scale, slowly varying mean component, and a small-scale, rapidly varying fluctuating component:

$$\boldsymbol{u} = \langle \boldsymbol{u} \rangle + \boldsymbol{u}', \quad \boldsymbol{B} = \langle \boldsymbol{B} \rangle + \boldsymbol{B}', \tag{1}$$

where the prime quantities represent the fluctuating part and the brackets $\langle \dots \rangle$ denote an intermediate averaging scale over which the fluctuating parts vanish, i.e., $\langle \boldsymbol{u}' \rangle = 0$ and $\langle \boldsymbol{B}' \rangle = 0$. Inserting Eq. (1) into the magnetohydrodynamical induction equation and applying this averaging operator yields:

$$\frac{\partial \langle \boldsymbol{B} \rangle}{\partial t} = \nabla \times (\langle \boldsymbol{u} \rangle \times \langle \boldsymbol{B} \rangle + \boldsymbol{\mathcal{E}} - \eta \nabla \times \langle \boldsymbol{B} \rangle), \tag{2}$$

where

$$\boldsymbol{\mathcal{E}} = \langle \boldsymbol{u}' \times \boldsymbol{B}' \rangle, \tag{3}$$

is the mean electromotive force (*emf*) due to the fluctuation of the flow and the magnetic field, and η is the magnetic diffusivity. The next step is to develop this *emf* in terms of the mean magnetic component and its derivatives. Because we are working here with vector fields, such a development is written as: