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Trajectory refinement of three-body orbits in the real solar system model

Diogene A. Dei Tos *, Francesco Topputo

Department of Aerospace Science and Technology, Politecnico di Milano, Via La Masa 34, 20156 Milan, Italy

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Abstract

In this paper, an automatic algorithm for the correction of orbits in the real solar system model is described. The differential equations governing the dynamics of a massless particle in the n-body problem are written as perturbation of the circular restricted three-body problem in a non-uniformly rotating, pulsating frame by using a Lagrangian formalism. The refinement is carried out by means of a modified multiple shooting technique, and the problem is solved for a finite number of trajectory states at several time instants. The analysis involves computing the dynamical substitutes of the collinear points, as well as several Lagrange point orbits, for the Sun–Earth, Sun–Jupiter, and Earth–Moon gravitational systems.

Keywords: n-body model; Dynamical system theory; Three-body problem; Solar system model

1. Introduction

In the circular restricted three-body problem (CRTBP), two gravitational attractions act simultaneously upon a massless particle. The CRTBP is the easiest extension of the two-body problem, and as such it allows reproducing solutions that depart from the conics. These range from Lagrange point orbits (Gómez et al., 2002a) to low-energy transfers (Topputo and Belbruno, 2015). Much effort has been put to characterise the region of the phase space about the equilibrium points (Jorba and Masdemont, 1999; Gómez and Mondelo, 2001). This is because most of the dynamics in the CRTBP can be related to that of the equilibrium points. Nevertheless, large differences in both position and velocity emerge when three-body orbits are integrated in the real solar system model (Luo et al., 2014). That is, since three-body orbits are typically defined in high-sensitive regions, where the gravitational attractions tend to balance, any additional term (ascribable to eccentricity of the primaries, fourth-body perturbations, or solar radiation pressure) causes large deviations between the CRTBP orbit and the real one (Luo and Topputo, 2015). Therefore, there is the need to devise methodologies to correct the three-body orbits, whilst still retaining their unique features.

The dynamical substitutes of the equilibrium points and families of periodic orbits are found by continuation in energy and period in Gómez et al. (2003) and Gómez and Mondelo (2001). Reduction to the centre manifold with enforcement of quasi-periodicity is performed in Gómez et al. (2002a), whereas a selection of some frequencies representing the main contributions of the solar system model is done in Gómez et al. (2002b). A large number of frequencies is instead considered in Hou and Liu (2011), where series expansion of the gravitational potential is carried out.

This paper further elaborates on an automated algorithm to refine three-body orbits in the real n-body problem, where the position of celestial bodies is modelled

* Corresponding author.
E-mail addresses: diogenealessandro.deitos@polimi.it (D.A. Dei Tos), francesco.topputo@polimi.it (F. Topputo).

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through precise ephemeris data. The differential equations governing the dynamics of a massless particle are written as perturbation of the CRTBP in a non-uniformly rotating and pulsating frame with a Lagrangian formalism (Gómez et al., 2002b). The refinement is carried out by means of a modified multiple shooting technique, and the problem is solved for the refined trajectory states at several locations. A finite set of NLP variables is used for the multiple shooting transcription. The obtained solution is then continued in a longer time domain through Fourier analysis and extrapolation. The generality of the algorithm lies in the possibility of handling both constrained and free boundary conditions. In the latter case, the problem is solved by minimising the correction at each step. The gradient of the objective function and the Jacobian of the constraints are computed and assembled in an automatic way. Families of halo, Lissajous, and planar Lyapunov orbits are reproduced, as well as dynamical substitutes of the Lagrange points.

The approach undertaken in this work possesses similarities with that in Lian et al. (2013) and Tang et al. (2013). Nonetheless, departures from previous works are featured in the implementation strategy, Fourier analysis, multiple shooting scheme, and performance index definition. These peculiarities are detailed hereafter. (1) The Fourier series approach used in this work is substantially different compared to the one used by Lian et al. (2013) and addressed in Gómez et al. (2002b). The perturbing frequencies are computed as the maxima of the signals Fourier transform (positions and velocities), instead of letting them be optimisation parameters for the trigonometric polynomial search. We thus show that it is not necessary to have very precise guesses for the extrapolation step, because the modified multiple shooting algorithm is able to converge and refine those orbits even with rougher frequency information on the perturbing bodies. This accelerates the whole procedure. (2) The amplitudes of the trigonometric polynomial are found here by minimising in the least squares sense the deviation between the real trajectory and the polynomial (see step iii). Conversely, a collocation method based on a refined Fourier analysis (described in Gómez et al. (2010a), Gómez et al. (2010b)) is used in Lian et al. (2013). Secondly, the algorithm is applied here to a wider range of cases. (3) Refined orbits have been computed for the Earth–Moon, Sun–Earth, and Sun–Jupiter problems. The results of the Earth–Moon problem are compared to the ones in Lian et al. (2013) and serve as solid benchmark to validate our procedure, and prove the algorithm correct and reliable. On the other hand, dynamical substitutes and refined quasi-periodic orbits of the Sun–Earth and Sun–Jupiter systems provide new scientific contribution on the subject. It is also demonstrated how the algorithm convergence properties do not depend on the particular mass ratio of the gravitational system and can be hence applied to a large variety of dynamical problems of this kind. (4) A minimisation procedure is coupled with a modified version of the multiple shooting, where the objective function is a quadratic form of the defects vector. The performance index thus accounts for the displacement between the initial and final orbit, not for this difference at each iteration. This implies that all the shooting legs are considered at the same time. On the other hand, a common parallel shooting at each iteration step has been implemented in Lian et al. (2013).

The remainder of the paper is organised as follows. In Section 2 the dynamical models are described. Effort is put in deriving the solar system n-body model. Section 3 details the algorithm for trajectory refinement. This is the core of the work: emphasis is put in the methodology and numerical procedure. The results are illustrated and discussed in Section 4, whereas final remarks are drawn in Section 5.

2. Dynamics

2.1. Roto-pulsating restricted n-body problem

The equations of the solar system restricted n-body problem are written as perturbation of the CRTBP, following the derivation in Gómez et al. (2002b). This makes it easier to retain the features of orbits derived in the CRTBP, and helps understanding the corrections applied in the refinement step. We avail ourselves of the JPL ephemeris DE430 (Folkner et al., 2014) to determine in a precise way the states of the Sun, the planets, and the Moon at given epochs in an inertial reference frame centred at the solar system barycentre (SSB).

Let $\boldsymbol{r}(t)$ and $\boldsymbol{v}(t)$ be the position and velocity, respectively, of a massless particle, P, in the inertial solar system barycentric frame, and let t be the dimensional time. Let also P_1 and P_2, of masses m_1 and m_2, $m_1 > m_2$, be the two primaries of the unperturbed CRTBP, and let $\mu = m_2/(m_1 + m_2)$ be their mass ratio. The aim is writing the equations of motion for P in a roto-pulsating frame (RPF), where P_1 and P_2 are at rest (see Fig. 1). We apply the transformation

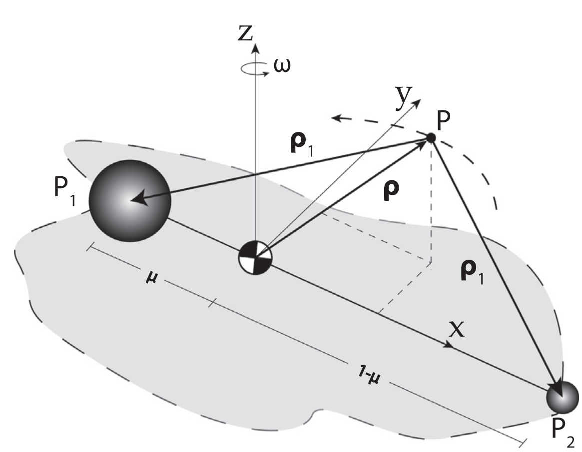

Fig. 1. Roto-pulsating reference frame.