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Statistical significance of spectral lag transition in GRB 160625B

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ABSTRACT

Recently Wei et al.[1] have found evidence for a transition from positive time lags to negative time lags in the spectral lag data of GRB 160625B. They have fit these observed lags to a sum of two components: an assumed functional form for intrinsic time lag due to astrophysical mechanisms and an energy-dependent speed of light due to quadratic and linear Lorentz invariance violation (LIV) models. Here, we examine the statistical significance of the evidence for a transition to negative time lags. Such a transition, even if present in GRB 160625B, cannot be due to an energy dependent speed of light as this would contradict previous limits by some 3-4 orders of magnitude, and must therefore be of intrinsic astrophysical origin. We use three different model comparison techniques: a frequentist test and two information based criteria (AIC and BIC). From the frequentist model comparison test, we find that the evidence for transition in the spectral lag data is favored at 3.05σ and 3.74σ for the linear and quadratic models respectively. We find that ΔAIC and ΔBIC have values $\gtrsim$ 10 for the spectral lag transition that was motivated as being due to quadratic Lorentz invariance violating model pointing to “decisive evidence”. We note however that none of the three models (including the model of intrinsic astrophysical emission) provide a good fit to the data.

1. Introduction

In special relativity, the speed of light, c, is constant and has the same value in all inertial frames of reference. However, this *ansatz* is no longer true in Lorentz violating standard-model extensions [2] and also several quantum gravity and string theory models (see [3,4] for reviews). In these models, Lorentz invariance is expected to be broken at very high energies close to the Planck scale, and the speed of light is dependent on the energy of the associated photon [5]. Although many astrophysical sources such as AGNs [6,7], pulsars [8] etc. have been used to search for LIV-induced light speed variation, most of these searches have been done with Gamma-Ray Bursts (GRBs) (See [9–16] and references therein). Results from searches for LIV prior to 2006 or so can be found in the reviews in [3,4]. We briefly enumerate some of the key results in the searches for this LIV since then.

Ellis et al.[9] considered a statistical sample of about 60 GRBs at a range of redshifts and modeled the observed time-lag as sum of a constant intrinsic offset and an additional offset due to energy-dependent speed of light. They found 4σ evidence that the higher energy photons arrive earlier than the lower energy ones. The estimated lower limit was about 0.9×10^{16} GeV. However, when an additional systematic offset was added to enforce the χ^2/DOF for the null hypothesis to be of order unity, the statistical significance reduced to about 1σ.

Abdo et al. [10] have used the detection of multi-GeV photons from a short GRB (GRB 090510), observed within a one-second window by Fermi-LAT to obtain a robust limit on the LIV scale of greater than the Planck scale. Vasileiou et al. [12] applied three complementary techniques on four GRBs from Fermi-LAT, and the most stringent limit they obtain is from GRB090510 of about 7.6 times the Planck scale for a linear Lorentz invariance violation and 1.3×10^{11} GeV for quadratic Lorentz invariance violation. These limits also rule out results from [14–16], who followed the same procedure of [9] and claimed evidence for a linear correlation between the LIV induced time lag and energy.

In contrast to the above searches, which looked for deterministic deviations in the speed of light as a function of energy, Vasileiou et al.[13] looked for stochastic deviations in the speed of light, using high energy observations of GRB090510 from Fermi-LAT, and obtained a limit on the quantum gravity scale of more than twice the Planck scale at 95% confidence level.

Most recently, Wei et al.[1] (W17) made an apparently convincing case pertaining to the evidence for a transition from positive to negative time lag in the spectral lag data for GRB 160625B, by using the data from Fermi-LAT and Fermi-GBM. By modeling the time

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lag as sum of intrinsic time-lag (due to astrophysical processes) and energy-dependent speed of light due to Lorentz invariance violation (LIV), which kicks in at high energies, they argued that this observation constitutes a robust evidence for a turnover in the spectral lag data. Subsequently, constraints on Lorentz invariance violation standard model extension coefficients have been obtained using this data [17]. However, no quantitative assessment of the observed statistical significance was made in these papers. In this work we compute the statistical significance by using three different model-comparison tests, namely frequentist hypothesis test, as well as information-criterion based tests.

The outline of this paper is as follows. We provide a succinct introduction to the model comparison techniques used, in Section 2. We briefly review the observations, data analysis and conclusions reached by W17 in Section 3. We then discuss the results from our model comparison tests using the same data in Section 4. Our conclusions can be found in Section 5.

2. Introduction to model comparison techniques

In recent years a number of both Bayesian and frequentist model-comparison techniques (originally developed by the statistics community) have been applied to a variety of problems in astrophysics, cosmology, and particle physics to address controversial issues. The aims of these techniques is two-fold. One is to find out which among the two hypothesis is favored. A second goal is to assess the statistical significance or *p*-value of how well the better model is favored. We note however that in many of these applications, not all the techniques used reach the same conclusions. Also the significances from the different techniques could be different. For our purpose, we shall employ multiple available techniques at our disposal to address how significant is the evidence for transition from positive to negative time lags in the spectral lag data. We briefly recap these techniques below. More details on each of these (from a physics/astrophysics perspective) can be found in various reviews [18–20].

- **Frequentist Test**: The first step in a frequentist model comparison test involves constructing a χ^2 between a given model and the data and then finding the best-fit parameters for each model. Then from the best-fit χ^2 and degrees of freedom, one calculates the goodness of fit for each model, given by the χ^2 probability or goodness of fit [21]:

$$P(\chi^2, \nu) = \frac{1}{2^{\frac{\nu}{2}}\Gamma(\nu/2)}(\chi^2)^{\frac{\nu}{2}-1}\exp\left(-\frac{\chi^2}{2}\right). \quad (1)$$

  where Γ is the incomplete Gamma function and ν is the total degrees of freedom.
  The best-fit model is the one with the larger value of χ^2 goodness of fit. If the two models are nested, then from Wilk's theorem [22], the difference in χ^2 between the two models satisfies a χ^2 distribution with degrees of freedom equal to the difference in the number of free parameters for the two hypotheses [18]. Frequentist tests have been used a lot in astrophysics, from testing claims of sinusoidal variations in G as a function of time [23] to classification of GRBs [24].
- **Akaike Information Criterion**: The Akaike Information Criterion (AIC) is used for model comparison, when we need to penalize for any additional free parameters to avoid overfitting. AIC is an approximate minimization of Kullback–Leibler information entropy, which estimates the distance between two probability distributions [20]. For our purpose, we use the first-order corrected AIC, given by [19]:

$$\mathrm{AIC} = \chi^2 + 2p + \frac{2p(p+1)}{N-p-1}, \quad (2)$$

  where N is the total number of data points and p is the number of free parameters. A preferred model in this test is the one with the smaller value of AIC between the two hypothesis. From the difference in AIC (Δ AIC), there is no formal method to evaluate a *p*-value.[1] Only qualitative strength of evidence rules are available depending on the value of ΔAIC [25].
- **Bayesian Information Criterion**: The Bayesian Inference Criterion (BIC) is also used for penalizing the use of extra parameters. It is given by [19]:

$$\mathrm{BIC} = \chi^2 + p\ln N. \quad (3)$$

  Similar to AIC, the model with the smaller value of BIC is the preferred model. The significance is estimated qualitatively in the same way as for AIC. Both AIC and BIC have been used for comparison of cosmological models [25–27].

Besides these techniques, the ratio of Bayesian evidence (or odds ratio) [28] has also been extensively used for model comparison in astrophysics and particle physics [26,28–30]. However, there have been criticisms regarding the usage of odds ratio for model comparison, since the Bayesian evidence depends on the priors chosen for the parameters [31,32]. We shall not consider Bayesian evidence in this work.

3. Summary of W17

W17 have used the spectral lag method to look for energy-dependent time lags in the arrival of photons of a particular GRB (namely GRB 160625B) using data from Fermi-LAT and Fermi-GBM, for which a remarkable transition from positive to negative time lags was observed in the arrival of higher energy photons. The observation of photons from the same source is aimed at providing tighter constraints on Lorentz invariance violation factor. We now briefly describe the *ansatz* made by W17 to fit the spectral lag data.

The observed time lags of photons of varying energies can be written down as :

$$\Delta t_{obs} = \Delta t_{int} + \Delta t_{LIV}, \quad (4)$$

where Δt_{int} is the intrinsic time lag between the emission of photon of a particular energy and the lowest energy photon from the GRB and Δt_{LIV} is the time-lag due to Lorentz invariance violation (hereafter, LIV). The uncertainty associated with Δt_{int} is the largest, as it depends upon the internal dynamics of the GRB itself which cannot be obtained from observations. W17 posited the following model for the intrinsic emission delay:

$$\Delta t_{int}(E)(\mathrm{sec}) = \tau\left[\left(\frac{E}{keV}\right)^{\alpha} - \left(\frac{E_0}{keV}\right)^{\alpha}\right], \quad (5)$$

where E_0=11.34 keV; whereas τ and α are free parameters. This functional form was based on the observation (from a recent study of the light curves of 50 GRBs), that most GRB light curves show positive time lags and the time tag is correlation with energy [33]. However, the analysis in [33] was only up to energies of 400 keV, which is well below the possible transition energy (found by W17) of $\sim$ 8 MeV. Therefore, there is no physics behind this particular functional form or evidence that this function describes the spectral lag for all GRBs. Therefore, it has no advantage over other functional forms, which may provide a comparable or even a better fit to the data. The remaining time lag has been attributed to the Lorentz violation effect, occurring at a considerably higher energy (closed to Planck scale) and can be written as [34]:

$$\Delta t_{LIV} = -\frac{1+n}{2H_0}\frac{E^n - E_0^n}{E^n_{QG,n}}\int_0^z \frac{(1+z')^n dz'}{\sqrt{\Omega_M(1+z')^3 + \Omega_\Lambda}}, \quad (6)$$

[1] See however [26] which posits a significance based on $\exp(-\Delta AIC/2)$.