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γ-ray telescopes using conversions to e^+e^- pairs: event generators, angular resolution and polarimetry

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ABSTRACT

We benchmark various available event generators in *Geant4* and *EGS5* in the light of ongoing projects for high angular-resolution pair-conversion telescopes at low energy. We compare the distributions of key kinematic variables extracted from the geometry of the three final state particles. We validate and use as reference an exact generator using the full 5D differential cross-section of the conversion process. We focus in particular on the effect of the unmeasured recoiling nucleus on the angular resolution. We show that for high resolution trackers, the choice of the generator affects the estimated resolution of the telescope. We also show that the current available generator are unable to describe accurately a linearly polarised photon source.

1. Introduction

γ-ray astronomy provides insight to understanding the non thermal processes in sources that undergo the most violent phenomena in the Universe, such as active galactic nuclei (AGN), gamma-ray bursts (GRB) and pulsars. Compton telescopes are mainly sensitive below photon energies of a few MeV, while existing pair telescopes are mainly sensitive above 100 MeV. In between lies an energy range in which no high-sensitivity measurements are available.

On the “pair side”, the main issue is the strong degradation of the angular resolution at low energy, which makes the rejection of the background noise from true photon conversions in the detector less efficient. Several technologies are being considered to improve the angular resolution such as silicon wafer stacks (i.e., without tungsten converters) [1–7], liquid noble-gas time projection chambers TPC (argon [8], or xenon [9]) and emulsions [10].

The two dominant contributions to the photon angular resolution are, on the one hand the single track angular resolution of the two leptons, and on the other hand the impossibility to measure the momentum of the recoiling ion. As the tracking performance improves, the missing recoil momentum becomes dominant. It is therefore necessary to describe it accurately in detector simulations.

Additionally, the conversion azimuthal angle of the electron and the positron with respect to the flight direction of the incident photon can in principle enable the measurement of the polarisation fraction and the polarisation angle of the source [11]. With the low-density detectors presently under development, the track directions can be measured with enough precision before multiple scattering ruins the azimuthal information carried by the pair [12]. The preliminary results of a recent experimental campaign on a particle accelerator beam are extremely encouraging in that regard [13].

As the technology improves, we see the necessity to validate the event generators used in detector simulations. In the present paper we reconsider the angular resolution due to the not measured recoil momentum with the new exact 5D event generator described in [14], without any approximation in the calculation of the angular resolution, extending the study to the lowest energies, very close to threshold, and providing 68%, 95% and 99.7% quantiles of the angular resolution. We then characterise the behaviour of other available event generators in *Geant4* and *EGS5*, through a few key kinematic variables of the pair conversion process. Finally, we examine the accuracy of these generators to describe the angular resolution and polarimetry potential for modern detectors with very fine tracking resolution.

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2. Kinematics

We use the indices $i = +, -, r, \gamma$ for the quantities related respectively to the positron, electron, recoil nucleus and photon.

- E_i, $\vec{p_i}$ are the energy and momentum of the particle.
- $\vec{u_i}$ is the propagation direction of the particle. By convention, we use $\vec{u_\gamma}$ as the z direction.
- $\varphi_i = \arctan\left(u_i^x / u_i^y\right)$ is the azimuthal angle of the particle.
- $x_i = E_i / E_\gamma$ is the fraction of photon energy carried away by the particle.
- $q = |\vec{p_r}|$ is the momentum transferred to the recoil nucleus.
- θ_{+-} is the pair opening angle, that is, the angle between the electron and positron momenta at conversion point,

$$\theta_{+-} = \arccos(\vec{u}_+ \cdot \vec{u}_-).$$

- θ is the angle between the incident photon and the observed direction $\vec{u}_{\text{pair}}$ obtained from the summed 4-vectors of the electron and positron. $\theta = \arccos u^z_{\text{pair}}$. We discuss in Section 4.3 the estimation of θ when the full 4-vector information is not available.
- ϕ, the conversion azimuthal angle, is defined as [15]

$$\phi = \frac{\varphi_+ + \varphi_-}{2}.$$

3. Validation of the simulation of the full 5D differential cross-section

With the prospect of having a pair telescope sensitive to photons of much lower energies than at present, there is a need for a conversion-to-pair event generator that is:

- exact down to threshold, that is, without any low-energy nor small-angle approximations,
- yielding a sampling of the true, five-dimensional (5D) differential cross section, that is, not a simple product of 1D projections,
- allowing the generation of conversion events by a linearly polarised beam.

We developed a generator [12], based on the BASES/SPRING event generator [16], that instantiate the VEGAS Monte Carlo method [17]: the differential cross section is tabulated in a 5-dimensional space on a grid that has been optimized to minimize its uncertainty. Events are then taken at random from the exact differential cross section using the acceptance-rejection method from the tabulated mock-up. Two methods were used to compute the 5D differential cross section. The Bethe-Heitler analytic expression [18] (that includes only the two dominant diagrams, an approximation that is valid for nuclear conversion and for high-energy triplet conversion), we then refer to it as the *Bethe-Heitler* model; A full diagram computation using the HELAS amplitude calculator [19], to which we refer as the *HELAS* model. The polarised form of the Bethe-Heitler 5D differential cross section is from Ref. [20], after a term was corrected by a factor of 2 [21]; the expressions that are used can be found in Ref. [15].

We confronted the calculations of that new generator to the analytic results on 1D projections published in the past and used it to study the performance of an actual TPC telescope and polarimeter [12].

The generator we developed relies on a relatively complex combination of packages. Our first step is to cross-validate the simulation by comparing three different approaches:

- A full simulation using Feynman diagram amplitudes calculated with HELAS, and 5D generator using BASES/SPRING.
- A simulation using the Bethe-Heitler approximation (very accurate in case the target is a nucleus much heavier than the electron), and 5D generator using BASES/SPRING. Bethe and Heitler neglected the diagrams for which the incident photon has a vertex with the ion (diagrams (c) and (d) of Fig. 1 of Ref.[12]).
- An analytic distribution of the recoil momentum of the nucleus obtained by Jost et al. [22] by integrating the BH differenttila cross section over other variables.

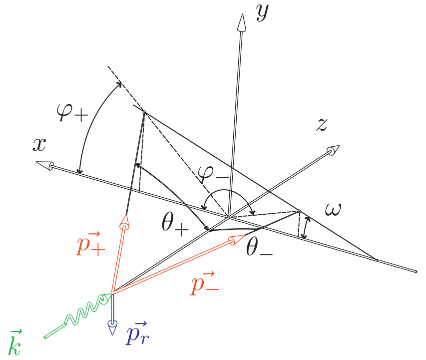

Fig. 1. Schema of a photon conversion.

We first compare the distributions of the recoil momentum q transferred to the nucleus. Since the recoil nucleus is cannot observed, q^2 is computed from the momenta p_γ, p_+ and p_- of the incident photon and of the produced leptons as: $q^2 = \left|\vec{p_\gamma} - (\vec{p_+} + \vec{p_-})\right|^2$. Fig. 2 shows an example of the distributions for various photon energies. The three distributions are perfectly consistent, which is a two-fold validation:

- a cross validation of the differential cross sections computed by HELAS and by the Bethe-Heitler expression;
- a validation of our use of the BASES/SPRING generator for gamma conversion.

As the q distributions extend over several orders of magnitude, we obtain an estimate of their magnitude from the containment value q_X, defined as the momentum value such that a given fraction X of the events have a recoil momentum smaller than q_X. We use the containment values $X = 68\%$, $X = 95\%$ and $X = 99.7\%$ that correspond approximately to 1σ, 2σ and 3σ for a Gaussian statistics. Fig. 3 shows the variation of q_X with E_γ for the three models. Once again we see that the three models are in perfect agreement over the considered energy range (< 1 GeV).

The recoil momentum q is crucial due to its close relation to the photon angular resolution σ_θ of the detector when the recoil nucleus cannot be measured as is the case for nuclear conversion. In that case, the direction is estimated from the electron-positron pair (sum of 4-vectors). At high energy, the angle between the photon and the pair can be approximated as $\theta \approx q/E_\gamma$ [14]. Fig. 3 right, shows that this approximation is exact above 40 MeV, and within 10% down to 1 MeV above the conversion threshold. A scaling factor of $E_\gamma^{1.25}$ is used to make the comparison easier [14].

Finally, we compare the distribution of the other kinematic variables x_+ and θ_{+-} for the *HELAS* and *Bethe-Heitler* models. The *Jost* model only describes q. We reduce the data by using the RMS of x_+, and the 68% containment value of θ_{+-}. Fig. 4 shows that over the full energy range, the two models are perfectly consistent. An examination of the full x_+ and θ_{+-} distributions confirms the agreement (plots not shown). At high energy, the distribution of x_+ is nearly flat between 0 and 1, which is reflected by the value of the RMS close to $1/\sqrt{12} \approx 0.29$.

We find that the approximation of the *Bethe-Heitler* model is in complete agreement with the full Feynman amplitude calculations