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Relativistic harmonics for turbulent wakefield diagnostics

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ABSTRACT

The propagation properties of relativistic harmonics excited in a plasma with an intense laser pulse is investigated theoretically and numerically. Focusing on the frequency separation, a cold electron fluid model in two spatial dimension is discussed to obtain the harmonic amplitude. The theoretical predictions are verified by performing particle-in-cell simulations in two spatial dimensions. When the laser amplitude is large, the strong ponderomotive force expels the electrons, creating a large amplitude density structures associated with the wakefield. The harmonics propagate obliquely with respect to the laser propagation direction, which is well represented by the structure of the high density layer resulting from the transverse poderomotive force. We also discuss a possible experimental setup to observe the density structures relevant to wakefield.

1. Introduction

The relativistic harmonic generation is an essential mechanism for the plasma based radiation sources and for the plasma based accelerators. In the presence of a linearly polarized electromagnetic wave in plasmas, electrons perform the well-known figure “8” motion. Using intense laser pulses, these collective currents can produce the coherent radiation of higher harmonics. A laser pulse propagating in an underdense plasma ($\omega_p/\omega_L < 1$, where ω_p and ω_L is the plasma and laser frequency, respectively) can excite the odd harmonics of the primal laser wave due to the density perturbation due to the figure eight motion and the nonlinear relativistic currents [1–9]. The amplitude of the harmonics obtained by their theoretical analyses shows scalable dependencies on the power of ω_p/ω_L. Thus the efficiency of the harmonic generation is considered to be relatively small in the underdense plasmas. The efficiency of the harmonic generation can be enhanced in overdense plasmas ($\omega_p/\omega_L > 1$) [10–14]. A laser pulse obliquely incident on the plasma slab with a sharp density gradient is evanescent and can be reflected back near the edge of the slab. The interaction between the laser pulse and the overdense plasma can excite the harmonic radiation propagating in the specular direction to the laser incidence.

Density gradient can be created in underdense plasmas when an intense laser pulse propagates. The ponderomotive force of intense laser pulse expels the electrons and form a wakefield, where some of the electrons can be accelerated up to high energies. The wakefield acceleration has been investigated to produce quasi-monoenergetic electrons [15–18], and recent years it has also been studied as a possible candidate of the extragalactic cosmic ray acceleration in astrophysical environments [19–26]. Importantly wakefield acceleration in the extreme astrophysical conditions can be scale down to the laboratory experiments with intense lasers [23,27,28], where we can directly observe the energy distribution functions of accelerated electrons. We have shown the universal power-law acceleration with an index of -2 due to the turbulent or incoherent wakefield, independent of laser and plasma conditions. In order to fully understand the acceleration process of energetic particles or cosmic rays in space, it is necessary to verify the relation between the observed distribution functions of particles and the waves in the plasma. So far, this has been only possible by in-situ observations in space plasmas. For the extragalactic cosmic rays, it is impossible to directly measure the local wakefield, and we had to rely on numerical simulations. The missing link is the wave observations. A coherent wakefield has been directly observed by [29]. It is very challenging to observe a turbulent or an incoherent wakefield. We will observe the density structures of electrons, which support the wakefield, by using the propagation property of the higher harmonics of the laser [9].

In this paper we discuss the relativistic harmonic generation in an underdense plasma by an intense linearly polarized laser pulse theoretically and numerically. In Section 2 we analytically discuss a cold electron fluid model in two spatial dimension when the laser electric field linearly polarized: either in the direction parallel to the 2-D system (p-polarization), or in the direction perpendicular to the system (s-polarization). Focusing on spatial and temporal variations, we separate the fast laser time scale from the slow plasma oscillation. We derive wave

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equations by further separating different orders of the laser frequency, then obtained the amplitude of the harmonics analytically. In Section 3, performing PIC (particle-in-cell) simulations in two spatial dimensions with a Gaussian laser pulse with either s- or p-polarization, the theoretical predictions are checked numerically. We simulate weakly relativistic cases for the s- and p-polarized laser pulse, respectively, then we consider a large amplitude laser pulse. In Section 4 we discuss the possible experiment to observe the density structure using the second harmonics.

2. Formulation

We consider a cold electron fluid.

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}) = 0, \tag{1}$$

$$\frac{\partial \vec{p}}{\partial t} + (\vec{v} \cdot \nabla)\vec{p} = -e\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right), \tag{2}$$

where n_e, $\vec{v}$, $\vec{p}$, and $-e$ represent the electron density, velocity, momentum, and charge, respectively. We assume ions to be static background since the scale length considered here is much smaller than the ion inertial length.

The electric $\vec{E}$ and magnetic $\vec{B}$ fields are given by Maxwell equations in the Coulomb gauge,

$$\vec{E} = -\frac{1}{c}\frac{\partial \vec{A}}{\partial t} - \nabla \Phi, \tag{3}$$

$$\vec{B} = \nabla \times \vec{A}, \tag{4}$$

$$\nabla \cdot \vec{A} = 0, \tag{5}$$

$$\frac{\partial^2 \vec{A}}{\partial t^2} - c^2 \triangle \vec{A} = 4\pi c \vec{j} - c\frac{\partial}{\partial t}\nabla \Phi, \tag{6}$$

where $\vec{A}$ and Φ are the vector and scalar potentials, respectively, and $\vec{j} \equiv = -e n_e \vec{v}$ is the current density. The Poisson's equation is written as

$$\triangle \Phi = -4\pi\rho, \tag{7}$$

where $\rho \equiv e(n_0 - n_e)$ is the charge density and n_0 is the ion density. Eq. (6) can be rewritten using Eq. (7) as

$$\frac{\partial^2 \vec{A}}{\partial t^2} - c^2 \triangle \vec{A} = 4\pi c \vec{j}_\perp, \tag{8}$$

where $\vec{j}_\perp$ is the transverse current density. We normalize all the quantities using, c, m, e, and $\omega_p = (4\pi n_0 e^2/m)^{1/2}$, such that $\omega = \omega_L/\omega_p$, $\vec{a} = e\vec{A}/(mc^2)$, $n = n_e/n_0$, $\varphi = e\Phi/(mc^2)$, $\vec{p}' = \vec{p}/mc$, and $\vec{r}' = \vec{r}\omega_p/c$. In this section " ' " will be omitted below.

We study laser-plasma interactions in two spatial dimensions, i.e., $\nabla = (\partial_x, \partial_y, 0)$, to compare with the numerical simulations. From the geometry, the z component of momentum equation $d_t(p_z - a_z) = 0$ shows the conservation of canonical momentum since $\partial_z = 0$. We consider a linearly polarized electromagnetic wave propagating in the x direction. The polarization of the laser electric field can be either y (p-polarization) or z direction (s-polarization). Such a laser field can be expressed as

$$a_1 = \sigma(x, y, t)\cos\phi, \tag{9}$$

where $\phi = kx - \omega t + \theta$, k and ω is the wave number and the angler frequency of the laser, respectively. Here we assume the slow variation in the field envelop, $\partial_x\sigma$, $\partial_y\sigma \ll k\sigma$, $\partial_t\sigma \ll \omega\sigma$. In this paper we focus on the time scale faster than the plasma oscillation and the envelop evolution.

2.1. s-polarization

First we consider the s-polarized laser pulse. For the fast variation, the wave equation of the z-component can be written as,

$$\partial_{xx}a_z - \partial_{tt}a_z = n\frac{a_z}{\gamma}, \tag{10}$$

where $\gamma = (1 + p_x^2 + p_y^2 + a_z^2)^{1/2}$ is the Lorentz factor of the electron. We expand the Lorentz factor as

$$\begin{aligned} \frac{1}{\gamma} &= \frac{1}{\gamma_0\left(1 + (p_x^2 + p_y^2 + \sigma^2\cos 2\phi/2)/\gamma_0^2\right)^{1/2}} \\ &= \frac{1}{\gamma_0} - \frac{1}{4\gamma_0^3}\sigma^2\cos 2\phi + O(a_z^4), \end{aligned} \tag{11}$$

where $\gamma_0 = (1 + \sigma^2/2)^{1/2}$. Note that $\sigma^2\cos 2\phi/(2\gamma_0^2) \le 1$ is true for arbitrary amplitude of laser pulse.

The momentum equations are written with the leading terms as

$$\begin{aligned} d_t p_x &= \partial_x\varphi - \frac{1}{2\gamma_0}\partial_x(a_z^2), \\ d_t p_y &= \partial_y\varphi - \frac{1}{2\gamma_0}\partial_y(a_z^2), \end{aligned} \tag{12}$$

where we neglect a_y since it is initially zero, and thus a_y cannot be the source of the forced oscillation of the electrons. Assuming the density perturbation $n = 1 + \delta n$,

$$\left(\partial_{tt} + \frac{1}{\gamma_0}\right)\delta n = \frac{1}{2\gamma_0^2}\partial_{xx}(a_z^2), \tag{13}$$

where we neglect the convection terms since they are higher order perturbations, and we assume that $\partial_{xx}\sigma$, $\partial_{yy}\sigma \ll k^2\sigma$. We obtain the second order density perturbation,

$$\delta n = \frac{-k^2\sigma^2}{(-4\omega^2 + 1/\gamma_0)\gamma_0^2}\cos 2\phi. \tag{14}$$

Thus the R.H.S. of Eq. (10) contains only odd harmonic sources. One can assume that $a_z = a_1 + a_3 + \cdots$ where a_1 and a_3 are respectively the parent and the third harmonic waves, and that

$$(\partial_{xx} - \partial_{tt})a_1 = \frac{a_1}{\gamma_0}, \tag{15a}$$

$$(\partial_{xx} - \partial_{tt})a_3 = \frac{a_3}{\gamma_0} - \frac{\sigma^3}{4\gamma_0^3}\cos 2\phi\cos\phi + \frac{\delta n\sigma}{\gamma_0}\cos\phi. \tag{15b}$$

From the first order Eq. (15a) one obtain the dispersion relation of electromagnetic wave in a plasma, using dimensional variables,

$$\omega_L^2 - k_L^2c^2 = \frac{\omega_p^2}{\gamma_0}, \tag{16}$$

where ω_L and k_L are the dimensional laser frequency and wavenumber, respectively. In general the third harmonic wavenumber k_3 is not equal to $3k$ because of the dispersion in Eq. (16) in the linear theory. Thus the phase velocity is different for different harmonics, resulting in the phase mismatch. The phase mismatch $\Delta\phi \propto (k_3 - 3k)x$ leads to the slow oscillation of the harmonic amplitude. We discuss the evolution of the wave equation in Appendix. Here we simply assume that $a_3 \equiv \sigma_3\cos 3\phi$ for the steady state value of the third harmonic amplitude since in Appendix the stationary point analysis gives the same result. From Eq. (15b) the third harmonic amplitude is written as

$$\sigma_3 = \frac{3}{64}\frac{\sigma^3}{\gamma_0^3}\frac{1}{-4\omega^2 + 1/\gamma_0}. \tag{17}$$