High Energy Density Physics 23 (2017) 95–114

Contents lists available at ScienceDirect

High Energy Density Physics

journal homepage: www.elsevier.com/locate/hedp

Nonrelativistic grey S_n-transport radiative-shock solutions

J.M. Ferguson[*,a], J.E. Morel[b], R.B. Lowrie[c]

[a] XCP-Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA
[b] Department of Nuclear Engineering, Texas A & M University, College Station, TX 77843, USA
[c] CCS-Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

ARTICLE INFO

Article History:
Received 19 December 2016
Revised 23 February 2017
Accepted 23 February 2017
Available online 24 February 2017

Keywords:
Radiation hydrodynamics
Variable Eddington factor
Radiative-shock solutions
Anti-diffusion

ABSTRACT

We present semi-analytic radiative-shock solutions in which grey S_n-transport is used to model the radiation, and we include both constant cross sections and cross sections that depend on temperature and density. These new solutions solve for a variable Eddington factor (VEF) across the shock domain, which allows for interesting physics not seen before in radiative-shock solutions. Comparisons are made with the grey nonequilibrium-diffusion radiative-shock solutions of Lowrie and Edwards [1], which assumed that the Eddington factor is constant across the shock domain. It is our experience that the local Mach number is monotonic when producing nonequilibrium-diffusion solutions, but that this monotonicity may disappear while integrating the precursor region to produce S_n-transport solutions. For temperature- and density-dependent cross sections we show evidence of a spike in the VEF in the far upstream portion of the radiative-shock precursor. We show evidence of an adaptation zone in the precursor region, adjacent to the embedded hydrodynamic shock, as conjectured by Drake [2,3], and also confirm his expectation that the precursor temperatures adjacent to the Zel'dovich spike take values that are greater than the downstream post-shock equilibrium temperature. We also show evidence that the radiation energy density can be nonmonotonic under the Zel'dovich spike, which is indicative of anti-diffusive radiation flow as predicted by McClarren and Drake [4]. We compare the angle dependence of the radiation flow for the S_n-transport and nonequilibrium-diffusion radiation solutions, and show that there are considerable differences in the radiation flow between these models across the shock structure. Finally, we analyze the radiation flow to understand the cause of the adaptation zone, as well as the structure of the S_n-transport radiation-intensity solutions across the shock structure.

Published by Elsevier B.V.

1. Introduction

In this paper we present semi-analytic, time-independent, 1D, planar, nonrelativistic, frequency-independent ("grey"), nonequilibrium, radiative-shock solutions using a variable Eddington factor (VEF) computed from angularly-discretized ("S_n") radiation transport (RT). Previous work by Sen and Guess [5], and Lowrie and Rauenzahn [6], presented semi-analytic nonrelativistic equilibrium-diffusion radiative-shock solutions, for which the material-radiation system is assumed to be in thermal equilibrium and the radiation field is linearly anisotropic. A linearly anisotropic radiation field implies that the radiation pressure is one-third of the radiation energy density which is associated with using a constant Eddington factor. Their work confirmed that continuous shock wave solutions existed and that radiation heat-conduction affected the material over a considerable distance ahead of the material shock into the radiation precursor. Semi-analytic, nonrelativistic, nonequilibrium-diffusion radiative-shock solutions were originally presented by Heaslet and Baldwin [7], and more recently by Lowrie and Edwards [1], which allow for the material and radiation temperatures to have separate values, but still assume that the radiation field is linearly anisotropic. However, the work by Heaslet and Baldwin neglected the radiation energy density and radiation pressure terms which were retained by Lowrie and Edwards. By separating the material and radiation temperatures those solutions provided a clearer understanding of the material response to the radiation, specifically, that the embedded hydrodynamic shock and the Zel'dovich temperature spike may exist independently of one another.

The work presented herein extends the work by Lowrie and Edwards [1] by modeling the radiation with S_n-transport in order to describe its angular dependence. This provides a better understanding of how the radiation flows through optically thick radiative shocks by allowing the Eddington factor to vary spatially across the shock, instead of using a constant Eddington factor. The idea of using S_n-transport to describe the radiation, for radiative-shock solutions, appears to have been first recommended in the conclusion of the paper by Sen and Guess [5], where they say, "It seems best to treat

* Corresponding author.
E-mail address: jmferguson@lanl.gov (J.M. Ferguson).

http://dx.doi.org/10.1016/j.hedp.2017.02.010
1574-1818/Published by Elsevier B.V.

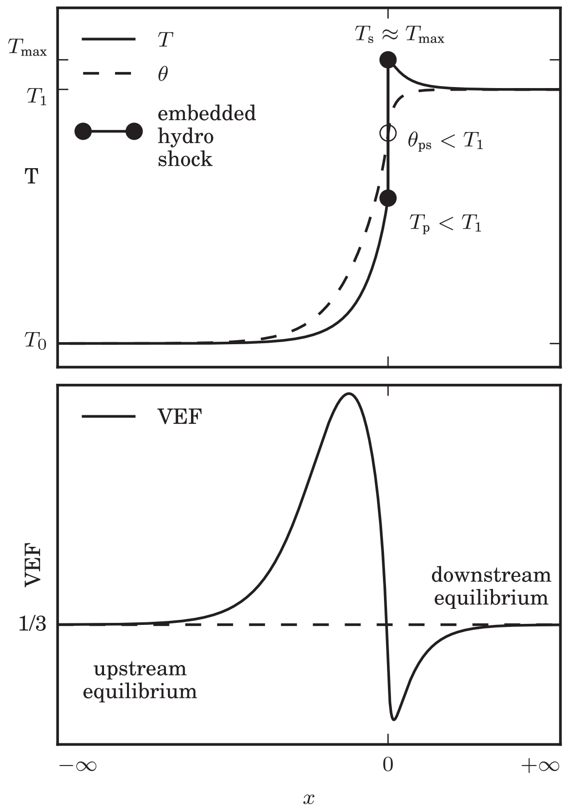

Fig. 1. A shock solution showing the material and radiation temperatures, T and θ, respectively, in the top plot, and the variable Eddington factor (VEF) in the bottom plot. The upstream ($x \approx -\infty$) and downstream ($x \approx +\infty$) equilibria regions of the shock are labeled, the unshocked material is traveling rightward into the shock, and the shock is traveling leftward. An embedded hydrodynamic shock exists between state-"p" and state-"s". The material temperature is discontinuous across the embedded hydrodynamic shock and labeled T_p and T_s, whereas the radiation temperature is continuous and labeled θ_{ps}.

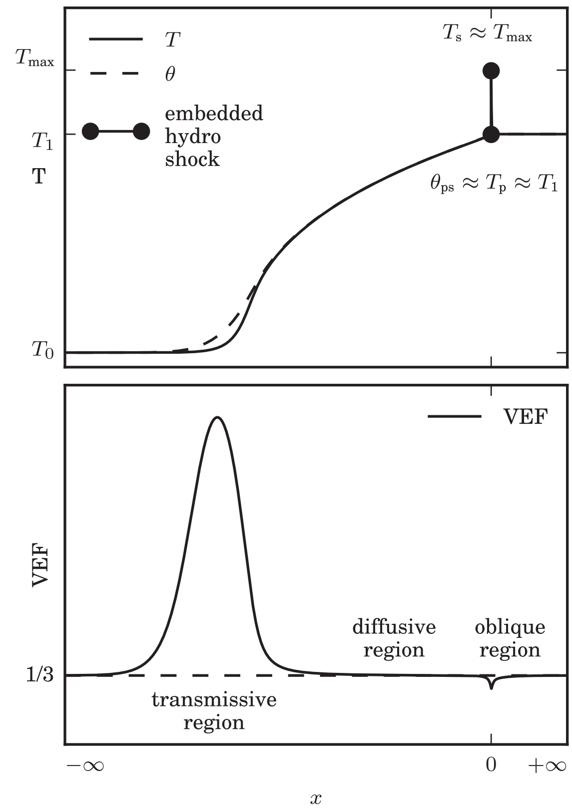

Fig. 2. A different shock solution than shown in Fig. 1. Here, the transmissive, diffusive, and oblique regions of the radiation flow are labeled. The transmissive region corresponds to the region where $f > 1/3$, and the radiation intensity reaches its maximum value at $\mu = -1$. The diffusive region corresponds to the region where $f \approx 1/3$ over a considerable distance, and the radiation is dominantly isotropic. The oblique region corresponds to the region where $f < 1/3$, and the radiation intensity reaches its maximum value near $\mu = 0$.

the radiation as a series of flux streams, in the manner of Chandrasekhar", who was the first to discretize the angular variable and integrate over it using quadrature methods [8]. Further, we verify conjectures made by Drake [2,3] that there should exist an adaptation zone adjacent to the embedded hydrodynamic shock, and subsequently that the temperatures very near the embedded hydrodynamic shock can take values that are greater than the value of the downstream equilibrium temperature. We also verify the prediction made by McClarren and Drake [4] that it is possible for the radiation energy density to be nonmonotonic under the Zel'dovich spike while the radiation flux is not near its equilibrium value. This set of ideas goes against the canonical literature for radiation hydrodynamics [9,10]. As a matter of practical utility, the solutions described here have already been used as a code-verification tool for a radiation-hydrodynamic (RH) code [11].

The solution method developed by Lowrie and Edwards [1] relies on the local Mach number being monotonic across the shock structure. Whether this is strictly true mathematically remains an open problem. We found no evidence that producing nonequilibrium-diffusion solutions violated this requirement for monotonicity. We did find that producing grey S_n-transport solutions may violate this monotonicity requirement in the precursor region. When the local Mach number becomes nonmonotonic it can cause our solution method to fail, although it is not a guarantee of failure.

The physical model used in this paper assumes that the system is optically-thick. As such, radiation cannot escape the material through either equilibrium boundary. Other authors have investigated other RH environments and solution methods. An analytic model of radiative shocks in a mixed, optically thick-thin environment was investigated by McClarren and co-authors [12]. Self-similar solutions via asymptotic analysis have been presented as an extension of the original Marshak solution by Lane and McClarren [13]. Other self-similar solutions have been produced by considering the method of Lie groups [14,15]. Asymptotic solutions of the second-kind, based on Barenblatt's work [16], were analyzed by Liang and Keilty [17]. Ion-electron shocks were recently studied by Masser, Wohlbier and Lowrie [18]. An initial attempt was made to study the effect of the radiation's frequency-dependence on the shock structure in the work by Holgado, Ferguson and McClarren [19]. In short, our particular solution method applies to a specific, theoretical RH environment, and care must be used when applying its analysis.

The rest of this paper is devoted to describing the solution method, presenting results obtained from it and comparing them with previous results, and analyzing specific features of the results. In Section 2, the necessary RH equations are collected, nondimensionalized, and reduced to steady-state ordinary differential equations (ODEs). In Section 3, the problem to be solved is defined. In Section 4, the nondimensionalized steady-state ODEs are manipulated into 1) two steady-state ODEs for the "RH solve", which is a two-point boundary value problem, and 2) "n" separate ODEs for the "RT solve", which represent "n" separate initial-value problems, and where there are "n" ODEs for S_n-transport. In Section 5, the results are presented, compared with previous results, and analyzed. In Section 6, we summarize our work and make recommendations for future work. In Appendix A, we nondimensionalize the ideal-gas γ-law equation-of-state used in this paper to show that the values of our nondimensional constants are consistent with the physics that we analyze. In Appendix B, we present the solution procedure.