



# Secular obliquity variations of Ceres and Pallas



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## ABSTRACT

We examine variations in the orientations of the orbit poles and spin poles of Ceres and Pallas, on time scales of a few million years. We consider these two bodies together because they have similar orbits, but very different present states of knowledge concerning internal mass distribution and spin pole orientation. For Ceres, the Dawn mission has recently provided accurate estimates of the current spin pole orientation, and the degree 2 spherical harmonics of the gravitational potential. The polar moment of inertia is not as well constrained, but plausible bounds are known. For Pallas, we have estimates of the shape of the body, and spin pole orientation and angular rate, all derived from optical light curves. Using those input parameters, and the readily computed secular variations in the orbit pole, we can compute long term variations in the spin pole orientation. This provides information concerning long term variations in insolation, which controls stability of surface volatiles.

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## 1. Introduction

The objective of this study is to examine long term behavior in the spin and orbit dynamics of two large asteroids, (1) Ceres and (2) Pallas. In particular, we focus on the motions of the spin and orbit poles. The orbit poles vary, in response to perturbations from the major planets, mainly Jupiter and Saturn, and precess about the invariable pole of the solar system. The spin poles precess about the respective orbit poles, in response to solar torques acting on the oblate figures, at rates which depend upon the oblateness of the mass distribution, and the ratio of the spin and orbit periods.

The obliquity, or angular separation between the spin and orbit poles, is particularly important for several reasons. One is that it controls the spatiotemporal patterns of diurnally averaged insolation (Ward, 1974) and thereby influences variations in surface and sub-surface temperature, which in turn determines stability of volatile compounds near the surface (Hayne and Aharonson, 2015; Schorghofer, 2016).

We consider Ceres and Pallas together, for several reasons. One is that their orbital periods, and semimajor axes are very similar, and thus the strength and cadence of orbital perturbations from Jupiter and other planets are quite similar, despite the fact that their current orbits are rather different, in terms of inclination and eccentricity. Another reason for considering them together is that current knowledge of the bodies is dramatically different. The

Dawn mission has measured the gravitational field and rotation state of Ceres very accurately (Park, 2016). In contrast, the shape and rotation of Pallas are much more poorly known, with both coming from light curve analyses, occultations, and images from HST and adaptive optics systems (Carry, 2010; Drummond, 2014; Schmidt, 2009). In this regard, Ceres and Pallas display a rather sharp contrast.

It will also emerge that departures from spherical symmetry in the shapes and mass distributions of these bodies are rather similar, with Ceres being only slightly closer to spherical than Pallas. As a result, the free precession periods of Ceres and Pallas are rather similar. The amplitude of their obliquity oscillation are different, but that is mainly due to different amplitudes of orbital inclination forcing.

We note that there have been some previous considerations of the spin pole dynamics of Ceres, including Bills and Nimmo (2011), Rambaux et al. (2011), Petit et al. (2014). Previous treatments of the spin dynamics of Pallas include Skoglov et al. (1996), Skoglov and Erikson (2002), and Lhotka et al. (2013).

Lhotka et al. estimated the present-day obliquity, and the spin pole precession rate of Pallas, but did not examine a history of obliquity variation. Skoglov and Erikson (2002) examined spin pole precession trajectories for 25 main belt asteroids, using reasonably accurate orbit models, and an assumed spin pole precession rate of 10 arcsec/year. The analysis of Skoglov et al. (1996) also assumed a spin pole precession rate for Pallas of 10 arcsec/year.

Taylor (1982) examined secular motion of the orbit of Pallas. His primary conclusion was that the 18:7 mean motion resonance

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between Pallas and Jupiter does not significantly influence the secular motion.

## 2. Coordinate systems

A peculiar feature of this study is that it requires consideration of 3 different coordinate frames, and both Cartesian and spherical coordinates in each frame. This arises from the differing historical conventions for reporting spin and orbit geometries.

Spin pole orientations of solar system bodies are almost always reported (Archinal, 2011) using spherical coordinates, in the celestial equatorial coordinate system (Urban and Seidelmann, 2012). Right ascension ( $\alpha$ ) is the angular distance, measured eastward along the celestial equator from the vernal equinox to the hour circle of the point in question. Declination ( $\delta$ ) is measured north or south of the celestial equator, along the hour circle passing through the point in question. Since Earth's equator plane precesses about the orbit plane, definition of an inertial coordinate system requires specification of a reference epoch. In most recent work, the epoch used is J2000, which is 1 January 2000. The Cartesian coordinates of the spin pole unit vector  $\hat{s}$ , in this reference frame, are given by

$$\hat{s} = \{\cos[\alpha] \cos[\delta], \cos[\alpha] \sin[\delta], \sin[\alpha]\} \quad (1)$$

Planetary orbital geometry is most often reported in terms of either Cartesian components of position  $P = \{x, y, z\}$  and velocity  $V = \{u, v, w\}$  vectors, or Keplerian elements. In either case, the reference frame is generally taken to have the ecliptic plane, and vernal equinox, at J2000, as defining the directions, and the position of the Sun defining the origin. The direction of the instantaneous orbit pole is given by the unit vector parallel to the orbital angular momentum vector

$$\hat{n} = \frac{P \times V}{|P \times V|} \quad (2)$$

Among the Keplerian elements, the inclination ( $I$ ) and longitude of ascending node ( $\Omega$ ) are relevant to orbit pole orientation. In fact, the orbit pole is given by

$$\hat{n} = \{\sin[I] \sin[\Omega], -\sin[I] \cos[\Omega], \cos[I]\} \quad (3)$$

We note that a unit vector, specified by spherical polar coordinates, consisting of longitude ( $\phi$ ) and latitude ( $\theta$ ) has the form

$$\hat{u}[\phi, \theta] = \{x, y, z\} = \{\cos[\theta] \cos[\phi], \cos[\theta] \sin[\phi], \sin[\theta]\} \quad (4)$$

The inverse of this transformation, from Cartesian back to spherical coordinates, is

$$\begin{aligned} \phi &= \arctan[x, y] \\ \theta &= \pi/2 - \arctan\left[z, \sqrt{x^2 + y^2}\right] \end{aligned} \quad (5)$$

The corresponding ecliptic frame longitude and latitude for the orbit pole, as given by (3), are

$$\begin{aligned} \phi_n &= \Omega - \pi/2 \\ \theta_n &= \pi/2 - I \end{aligned} \quad (6)$$

The obliquity ( $\varepsilon$ ), or angular separation between spin and orbit poles is given by

$$\cos[\varepsilon] = \hat{n} \cdot \hat{s} \quad (7)$$

However, the unit vectors must first be written in a common coordinate frame. The versions listed above, in Eqs. (1) and (3) are in different frames. To convert a vector from the equatorial to the ecliptic frame, we rotate about the x-axis, through an angle equal to Earth's obliquity (Hilton, 2006)

$$\varepsilon^* = 23.43928108^\circ \quad (8)$$

**Table 1**  
Orbital periods.

Body	Period day	Period year
Ceres	1681.2435	4.602994
Pallas	1684.9040	4.613016
Jupiter	4332.3548	11.86134

The corresponding x-axis rotation matrix, for rotation through a generic angle  $q$ , has the form

$$R_1[q] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & +\cos[q] & -\sin[q] \\ 0 & +\sin[q] & +\cos[q] \end{bmatrix} \quad (9)$$

The conversion thus has the form

$$\begin{bmatrix} x_{equ} \\ y_{equ} \\ z_{equ} \end{bmatrix} = R_1[\varepsilon^*] \cdot \begin{bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{bmatrix} \quad (10)$$

and, of course, the inverse transformation is just

$$\begin{bmatrix} x_{ecl} \\ y_{ecl} \\ z_{ecl} \end{bmatrix} = R_1[-\varepsilon^*] \cdot \begin{bmatrix} x_{equ} \\ y_{equ} \\ z_{equ} \end{bmatrix} \quad (11)$$

In our consideration of orbital and rotational variations of Ceres and Pallas, we will specify the present day orbit geometry in terms of inclination ( $I$ ) and nodal longitude ( $\Omega$ ), while the present spin poles are given either in terms of right ascension ( $\alpha$ ) and declination ( $\delta$ ), or ecliptic longitude ( $\lambda$ ) and latitude ( $\beta$ ). For an orbit pole  $\hat{n}$  specified by  $\{I, \Omega\}$ , and spin pole  $\hat{s}$  specified by  $\{\lambda, \beta\}$ , the obliquity is given by

$$\cos[\varepsilon] = \hat{n} \cdot \hat{s} = \cos[I] \sin[\beta] - \sin[I] \cos[\beta] \sin[\lambda - \Omega] \quad (12)$$

This calculation is relatively simple because both sets of coordinates are given in the same (ecliptic) frame. If the spin pole is given in terms of right ascension and declination, the calculation is somewhat more complicated, since there is an additional transformation from equatorial to ecliptic frames.

In consideration of long term evolution of orbits, using secular variation models (Brouwer and van Woerkom, 1950; Knezevic, 1986; Laskar, 1988), for orbital evolution, it is common (though not necessary) to use the invariable plane of the solar system, in place of the ecliptic plane. The invariable plane is, by definition, perpendicular to the solar system angular momentum vector, and is close to Jupiter's orbit plane (Souami and Souchay, 2012).

The advantage of using the invariable plane, in secular variation models, is that it makes the motions appear somewhat simpler. However, since the orbital element initial conditions are most often specified in terms of ecliptic elements, we will use that frame for our secular models. In particular, we use the ecliptic and equinox of J2000.

## 3. Shorter period effects

In this paper we will mainly be examining secular variations in the orbital elements of Ceres and Pallas. We believe that our secular variation model accurately reproduces behavior on time scales from  $10^3$  to  $10^6$  years, or perhaps even somewhat longer. However, due to a near 18:7 mean motion resonance with Jupiter, both Ceres and Pallas experience significant orbital perturbations on shorter time scales. In the discussion below, we use orbital elements obtained from the JPL Horizons web site, spanning 900 years, from 1600 to 2500 CE. Over that time span, the mean orbital periods of Ceres, Pallas, and Jupiter are given in Table 1. The JPL Horizons web site can be used to obtain osculating Keplerian elements. The

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