ARTICLE IN PRESS
JID: YICAR [m5G;September 15, 2016;20:28]

Icarus 000 (2016) 1–15

Contents lists available at ScienceDirect

Icarus

journal homepage: www.elsevier.com/locate/icarus

Numerically modelling tidal dissipation with bottom drag in the oceans of Titan and Enceladus

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ARTICLE INFO

Article history:
Available online xxx

Keywords:
Satellites
Ocean tides
Bottom drag
Titan
Enceladus

ABSTRACT

Icy satellites that contain subsurface oceans require sufficient thermal energy to prevent the liquid portion of their interiors from freezing. We develop a numerical finite difference model to solve the Laplace Tidal Equations on a sphere in order to simulate tidal flow and thermal energy dissipation in these oceans, neglecting the presence of an icy lid. The model is applied to Titan and Enceladus, where we explore how Rayleigh (linear) and bottom (quadratic) drag terms affect dissipation. The latter drag regime can only be applied numerically. We find excellent agreement between our results and recent analytical work. Obliquity tide Rossby-wave resonant features become independent of ocean thickness under the bottom drag regime for thick oceans. We show that for Titan, dissipation from this Rossby-wave resonance can act to dampen the rate of outward orbital migration by up to 40% for Earth-like values of bottom drag coefficient. Gravity-wave resonances can act to cause inward migration, although this is unlikely due to the thin oceans required to form such resonances. The same is true of all eccentricity tide resonances on Enceladus, such that dissipation becomes negligible for thick oceans under the bottom drag regime.

1. Introduction

Thermal energy in the interiors of outer Solar System icy satellites is supplied primarily by radiogenic decay and tidal dissipation. Radiogenic decay plays a role in large icy satellites with a significant portion of silicate material in their interiors (Hussmann et al., 2006). This role, however, diminishes with decreasing mass of silicate material. Small satellites have a high surface area to volume ratio, and consequently thermal energy generated from radioactive decay is lost on a timescale much less than the age of the Solar System. Yet, several small and medium sized icy satellites have confirmed global oceans, suggesting greater interior heating than that provided by radiogenic decay alone.

This work focuses on Titan and Enceladus. Several interior models and lines of evidence suggest Titan contains a subsurface ocean (Baland et al., 2014; Bills and Nimmo, 2011; Iess et al., 2012; Mitri et al., 2014; Sohl et al., 2003; 2014). Enceladus also shows strong evidence of a liquid ocean beneath its surface. Originally it was thought that this liquid reservoir was localised beneath the South Polar Terrain (SPT) of the satellite (e.g., Collins and Goodman, 2007). However, recent modelling of the degree-2 gravity field was consistent with a global ocean with greatest thickness beneath the SPT, although such models are non-unique (Iess et al., 2014; McKinnon, 2015). Most recently, the large forced libration of Enceladus indicates a decoupling of the icy shell from its interior, and thus the presence of a global ocean (Thomas et al., 2016).

This paper is intended to introduce and verify a new numerical model for solving thin shell fluid dynamics in planetary bodies. The model is therefore useful for investigating ocean dissipation in a variety of icy satellites. Assumptions made in the development of the numerical code reflect those made in the semi-analytical models to which our results are compared. This ensures the most accurate verification of the numerical model. We also introduce bottom drag into the model, an extension that is only possible numerically and in simplified scaling analysis (Chen et al., 2014). We explore this drag regime for oceans on Titan and Enceladus.

1.1. Tidal dissipation

Any satellite that passes through a varying gravitational potential will experience some form of tidal dissipation. The time varying gravitational potential may be a result of the satellite's orbital eccentricity and/or obliquity, as well as any non-synchronous rotation. For a satellite in (near) synchronous rotation, the gravitational tidal potential will vary periodically over the satellite's orbit. The changing potential does mechanical work on the satellite, and a portion of this work is converted to thermal energy. This process

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http://dx.doi.org/10.1016/j.icarus.2016.09.022
0019-1035/

Please cite this article as: H.C.F.C. Hay, I. Matsuyama, Numerically modelling tidal dissipation with bottom drag in the oceans of Titan and Enceladus, Icarus (2016), http://dx.doi.org/10.1016/j.icarus.2016.09.022

is known as tidal dissipation. As long as sufficient orbital or rotational energy remains in the system (in the form of eccentricity, obliquity or non-synchronous rotation), tidal dissipation will occur.

Both the solid and fluid regions of a satellite will experience tidal dissipation. Despite this, and the overwhelming evidence for and abundance of subsurface oceans in the icy satellites, the majority of dissipation studies have focused on only solid-body tides, (e.g., Moore and Schubert, 2000; Tobie et al., 2005; Roberts and Nimmo, 2008; Beuthe, 2013). Most terrestrial tidal dissipation occurs within the oceans, and while Earth has a complex dynamic between tidally-induced ocean flow and its continents, it illustrates the importance of considering ocean dissipation.

The effect of ocean dissipation in outer planet satellites was first considered for Titan by Sagan and Dermott (1982), who analytically derived expressions to estimate dissipated energy in a global hydrocarbon surface ocean. In doing so they attempted to explain Titan's relatively high eccentricity. The same problem was then approached numerically by Sears (1995), who solved the Laplace Tidal Equations to derive time averaged estimates of dissipated energy within a hydrocarbon ocean of varying thicknesses. Sears' numerical model is the basis for the model described in this paper.

More recently, Tyler (2008, 2009, 2011, 2014) has done extensive work on ocean dissipation, showing that thermal energy released through interior fluid motions can theoretically prevent a subsurface liquid from freezing. By exploring how ocean thickness and drag coefficient affect dissipation, Tyler (2011) discovered ocean dissipation resonances. These resonances tend to occur for oceans of a particular thickness, where oceanic planetary waves resonantly interact with the periodic tidal forcing, allowing enhanced tidal flow and consequently significant tidal dissipation. Matsuyama (2014) developed a similar model to that used by Tyler (2011), adding the effects of ocean loading, self-attraction, and deformation of the solid regions. These effects were shown to alter the position and magnitude of these dissipative resonances. Kamata et al. (2015) modelled the effects of an icy shell on Love number resonances, but did not include any ocean dynamics.

Tyler (2011), Matsuyama (2014), and Chen et al. (2014) all considered Rayleigh (linear) drag in their models. Chen et al. (2014) also developed a set of scaling laws to model ocean dissipation in the bottom (quadratic) drag regime. The numerical model presented in this work is capable of solving the LTEs using both Rayleigh and bottom drag, as is typical in terrestrial ocean dissipation studies (Egbert and Ray, 2001; Jayne and Laurent, 2001; Jeffreys, 1921; Taylor, 1920; Zahel, 1977). In Sears (1995), a very complex approach was used in order to study ocean dissipation on Titan. Three drag terms were included simultaneously in his model: Rayleigh and bottom drag, as well as eddy induced viscosity. Each one of these relies on unknown coefficients, for which Sears (1995) chose one value for each. This greatly over-constrains his results. Here we employ a far more idealised approach, specifically investigating only Rayleigh or bottom drag in sequence over a vast parameter space. This allows us to understand how each of these drag models affect ocean dissipation. These two drag regimes are briefly described below.

1.2. Rayleigh drag

A linear formulation of drag was first introduced as the Guldberg-Mohn approximation of *virtual* internal friction in 1876 (Neumann, 1968). Now known as Rayleigh drag, the approximation describes drag within a fluid that is proportional and opposite to the fluid's velocity. That is, the drag force per unit mass $\boldsymbol{F_d} = -\alpha \boldsymbol{u}$, where α is some drag time scale known as the coefficient of Rayleigh drag with units of s^{-1}. The flow velocity is $\boldsymbol{u}$.

Rayleigh drag can be thought of as a macroscopic description of drag between adjacent fluid elements in a moving liquid.

1.3. Bottom drag

Terrestrial ocean dissipation studies often employ a drag model that scales with the square of the fluid's velocity. This quadratic dependence of drag on flow velocity is referred to as *bottom drag* (Gill, 1982), and arises due to turbulent flow interacting with some bottom boundary, such as the ocean floor. Large tangential shear stresses associated with this interface generate a turbulent boundary layer where there is a significant transfer of momentum from the flow. While such turbulence cannot be resolved at the scale of planetary ocean simulations, the bottom drag coefficient c_D is empirically derived to include the frictional effect of this turbulence at the planetary scale.

In this work we investigate the effects of bottom drag on ocean dissipation for both Titan and Enceladus, structuring the paper as follows. Firstly, we introduce our numerical method in Section 2, describing the grid structure and numerical solver as well as its current limitations. We then apply this model to both Titan (Section 3) and Enceladus (Section 4) in turn for each drag model, examining how dissipation differs between each case. The Rayleigh drag results are compared to semi-analytical solutions of Matsuyama (2014). We also compare the bottom drag results to scaling laws developed by Chen et al. (2014).

2. Methodology

This section describes some of the theory and methods involved in this work. The governing equations of the numerical model and their applicability are discussed in Sections 2.1 and 2.2. We then provide analytical expressions for the degree-2 tidal potential in Section 2.3. Following this, descriptions of the discretisation and numerical scheme are outlined in Section 2.5, with a summary of our simulations in Section 2.6.

2.1. Laplace tidal equations

The equations of motion and continuity that describe ocean tidal flow in the shallow water limit are known as the Laplace Tidal Equations (LTEs) (Lamb, 1932). The main assumption leading to this set of equations is that radial (vertical) ocean flow is negligible when compared to lateral flow, reducing the problem to two dimensions. This is indeed a good approximation at the planetary scale, where lateral flow length scales span significantly greater distances than the thickness of an ocean. The conservation of mass (Eq. (1)) and momentum (Eq. (2)) that make up the LTEs, including both Rayleigh and bottom drag, are given as (Matsuyama, 2014; Sears, 1995; Tyler, 2008):

$$\partial_t \eta + \nabla \cdot (h\boldsymbol{u}) = 0, \tag{1}$$

$$\partial_t \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} + \alpha \boldsymbol{u} + \frac{c_D}{h}|\boldsymbol{u}|\boldsymbol{u} + g\nabla\eta = (1 + k_2 - h_2)\nabla U_2 \,. \tag{2}$$

Eq. (1) consists of two terms. The first is the time rate of change of vertical sea surface displacement, η, about some equilibrium level, h_0, where the total ocean thickness, $h = h_0 + \eta$. The second term is the divergence of the ocean thickness multiplied by the surface velocity vector, $\boldsymbol{u} \equiv (u, v)$, where u and v are the eastward and northward velocity components, respectively. Clearly, mass divergence and convergence is balanced by the vertical motion of the ocean free surface.

The term on the right hand side of Eq. (2) is an applied force per unit mass. ∇U_2 is the gradient of the degree-2 tide raising potential, discussed in Section 2.3. It is multiplied by Love's reduction factor, $1 + k_2 - h_2$. Love's first number, k_2, is a proportionality