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# Onset of oligarchic growth and implication for accretion histories of dwarf planets

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ABSTRACT

We investigate planetary accretion that starts from equal-mass planetesimals using an analytic theory and numerical simulations. We particularly focus on how the planetary mass $M_{\rm oli}$ at the onset of oligarchic growth depends on the initial mass $m_0$ of a planetesimal. Oligarchic growth commences when the velocity dispersion relative to the Hill velocity of the protoplanet takes its minimum. We find that if $m_0$ is small enough, this normalized velocity dispersion becomes as low as unity during the intermediate stage between the runaway and oligarchic growth stages. In this case, $M_{\rm oli}$ is independent of $m_0$. If $m_0$ is large, on the other hand, oligarchic growth commences directly after runaway growth, and $M_{\rm oli} \propto m_0^{3/7}$. The planetary mass $M_{\rm oli}$ for the solid surface density of the Minimum Mass Solar Nebula is close to the masses of the dwarf planets in a reasonable range of $m_0$. This indicates that they are likely to be the largest remnant planetesimals that failed to become planets. The power-law exponent $q$ of the differential mass distribution of remnant planetesimals is typically −2.0 and −2.7 to −2.5 for small and large $m_0$. The slope, $q \simeq -2.7$, and the bump at $10^{21}$ g (or 50 km in radius) for the mass distribution of hot Kuiper belt objects are reproduced if $m_0$ is the bump mass. On the other hand, small initial planetesimals with $m_0 \sim 10^{13}$ g or less are favored to explain the slope of large asteroids, $q \simeq -2.0$, while the bump at $10^{21}$ g can be reproduced by introducing a small number of asteroid seeds each with mass of $10^{19}$ g.



## 1. Introduction

The standard scenario of planet formation begins with small dust floating in a gaseous disk. Clumping of dust particles either due to streaming instability (Johansen et al., 2015b; 2007), turbulent concentration (Chambers, 2010; Cuzzi et al., 2010; 2008; Hopkins, 2016), or direct sticking (Garaud et al., 2013; Kataoka et al., 2013; Okuzumi et al., 2012; Windmark et al., 2012) leads to formation of gravitationally bound planetesimals. These planetesimal formation models generally predict the size of initial planetesimals ranging from 10 to 100 km or even larger although some direct sticking models (Garaud et al., 2013; Windmark et al., 2012) predict planetesimals as small as 100 m. The initial planetesimal size implied from planetary accretion models is controversial. For models of asteroid formation, Morbidelli et al. (2009) suggested large size ( ∼100 km) of initial planetesimals whereas (Weidenschilling, 2011) showed that the asteroid size distribution can also be reproduced from initially small ( ∼100 m) planetesimals. Accretion models for Kuiper belt objects (KBOs) generally favor small initial planetesimals (1–10 km) since the accretion timescale is very long for initially large planetesimals (Kenyon and Bromley, 2012; Schlichting et al., 2013).

Once planetesimals form, they further grow via mutual collisions. In the early stage of planetary accretion, called the runway growth stage, large planetesimals grow quickly due to their gravitational focusing effects while small planetesimals nearly remain at their initial sizes (Barnes et al., 2009; Greenberg et al., 1978; Kokubo and Ida, 1996; Wetherill and Stewart, 1989). As growth proceeds, the largest bodies, often called planetary embryos or protoplanets, start to dominate the gravitational scattering effect, i.e. viscous stirring (Ida and Makino, 1993). As the number of protoplanets decreases due to mutual merging, their orbital separation increases. Eventually, each small body is predominantly stirred by a single protoplanet (Kokubo and Ida, 1998). Because of the viscous stirring effect of each protoplanet, among protoplanets, larger ones grows more slowly than smaller ones. As a result, protoplanets with similar masses grow at a similar rate (Kokubo and Ida, 1998; 2002). This growth mode is called oligarchic growth. Although the basic pictures of runaway and oligarchic growth are

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apparently well known, the pathway from runaway growth to oligarchic growth described in the literature varies from author to author. The mass of a protoplanet $M_{\rm oli}$ at the onset of oligarchic growth is not well understood either.

Based on *N*-body simulations and the analytic formulation, Ida and Makino (1993) derived the condition that protoplanets rather than small planetesimals dominate viscous stirring. They argued that once this condition is fulfilled, transition from runaway growth to oligarchic growth occurs. As pointed out by Ormel et al. (2010b), however, this condition is already satisfied even during runaway growth and is thus necessary but not sufficient for oligarchic growth.

Kokubo and Ida (1998) performed direct *N*-body simulations of oligarchic growth and found that the orbital separation of neighboring protoplanets is about 10 Hill radii as a result of orbital repulsion. Since this width is the typical scale of gravitational influence of a protoplanet, this single protoplanet dominates viscous stirring in the region of its gravitational influence. In earlier stages of planetary accretion, the mutual separation between neighboring protoplanets is smaller than 10 Hill radii and their regions of gravitational influence mutually overlap.

Ormel et al. (2010b) derived a new condition for the transition from runaway to oligarchic growth. They examined evolution of the velocity dispersion $u$ of small planetesimals normalized by the Hill velocity $v_{\rm H}$ of the largest protoplanet, where $v_{\rm H}$ is the product of Hill radius of the protoplanet and the Keplerian frequency. The normalized velocity $u/v_{\rm H}$ decreases during the runaway growth while it increases during oligarchic growth. The transition between these two stages then occurs when $u/v_{\rm H}$ takes its minimum. Based on timescale arguments for the dispersion dominant regime ($u/v_{\rm H} > 1$), they derived the planetary mass $M_{\rm oli}$ at the transition. They showed that $M_{\rm oli} \propto m_0^{3/7}$, where $m_0$ is the initial mass of each planetesimal. Morishima et al. (2013) derived a similar transition mass without timescale arguments assuming that the protoplanet is the largest body of a continuous mass distribution with the power-law exponent $q$ of $-2.5$, which is the typical value during runaway growth (Kokubo and Ida, 1996; Morishima et al., 2013; Ormel et al., 2010a).

Using the simple analytic formulation originally developed by Goldreich et al. (2004), Lithwick (2014) showed that runaway growth is followed by a new stage – the trans-Hill stage – during which $u/v_{\rm H}$ remains nearly unity. The growth rates of protoplanets during the trans-Hill stage are independent of mass and that leads to $q \simeq -2.0$ for the massive side of the mass distribution. The trans-Hill stage is terminated by emergence of oligarchic growth, in which the mutual separation between protoplanets is scaled by the Hill radius (Kokubo and Ida, 1998). The planetary mass $M_{\rm oli}$ at the onset of oligarchic growth subsequent to the trans-Hill stage is independent of $m_0$. The concept of the trans-Hill stage was confirmed by coagulation simulations of Shannon et al. (2015).

Schlichting and Sari (2011) also discussed the transition stage after runaway growth using the analytic formulation of Goldreich et al. (2004). They also found that $u/v_{\rm H}$ takes a fixed value close to but somewhat larger than unity during the transition stage, contrary to Lithwick (2014). The difference comes from their assumptions; while Schlichting and Sari (2011) assumed that growth of protoplanets is equally contributed by merging with small planetesimals and mutual merging between protoplanets (equal accretion), Lithwick (2014) assumed that growth of protoplanets is dominated by merging with small planetesimals. Lithwick (2014), in fact, analytically proved his assumption and criticized the unjustified assumption employed by Schlichting and Sari (2011).

As reviewed, different authors showed apparently different conditions for or pathways to the onset of oligarchic growth. The objective of the present paper is to establish a comprehensive picture for the onset of oligarchic growth and to derive $M_{\rm oli}$ for arbitrary $m_0$, using an analytic formulation and numerical simulations. Particularly, we will clarify the following three points.

1. Three conditions for the onset of oligarchic growth were indicated in the literature:
   (a) protoplanets dominate viscous stirring,
   (b) the mutual separation between neighboring protoplanets is 10 Hill radii, and
   (c) $u/v_{\rm H}$ takes its minimum.
   The condition (b) is equivalent to the definition of oligarchic growth. We will show that the condition (c) is useful to pin down the timing of the onset of oligarchic growth, as shown by Ormel et al. (2010b), although the condition (a) is also required for oligarchic growth.
2. Ormel et al. (2010b) and Morishima et al. (2013) showed that both $M_{\rm oli}$ and the minimum $u/v_{\rm H}$ depend on $m_0$ while Schlichting and Sari (2011) and Lithwick (2014) showed that $M_{\rm oli}$ and the minimum $u/v_{\rm H}$ are both independent of $m_0$. We will show that the formulations of Ormel et al. (2010b) and Morishima et al. (2013) are applicable for large $m_0$ while the formulations of Schlichting and Sari (2011) and Lithwick (2014) are applicable for small $m_0$. The critical value of $m_0$ which separates these two regimes will be explicitly formulated. The transition stage between the runway growth and oligarchic growth stages appears only if $m_0$ is lower than this critical value.
3. Schlichting and Sari (2011) and Lithwick (2014) employed different assumptions for the contribution of mutual merging between protoplanets to their growth. We will show that as long as their equations are employed, growth of protoplanets is dominated by merging with small planetesimals, as proved by Lithwick (2014). We will, however, show that the contribution of mutual merging can be comparable to or even larger than the small bodies' contribution due to very small inclinations of large bodies and to small bodies' velocities somewhat larger than the Hill velocity of large bodies.

Besides clarifying the picture of the onset of oligarchic growth, at least, the following two important implications are discussed from $M_{\rm oli}$. First, the masses of the dwarf planets are potentially close to $M_{\rm oli}$. Protoplanets more massive than $M_{\rm oli}$ have low velocity dispersion due to dynamical friction of surrounding planetesimals. As a result, they efficiently merge together and form large planets, while leaving remnant planetesimals with the largest one about $M_{\rm oli}$ in mass (Morishima et al., 2008). This argument is probably applied to asteroids. For KBOs at large heliocentric distances, planetary accretion is likely to be incomplete. Recent observations of KBOs (Adams et al., 2014; Fraser et al., 2014) showed that the largest two bodies, Pluto and Eris, are distinctively separated from a continuous size distribution, possibly indicating that they actually entered into oligarchic growth but their growth was stalled immediately after that. In either case, it is worthwhile to compare the masses of the dwarf planets in the asteroid and Kuiper belts with $M_{\rm oli}$ as it helps us to infer the physical parameters of the proto-solar nebula. In 2015, NASA's Dawn and New Horizons spacecraft arrived at Ceres (Russell et al., 2015) and Pluto (Stern et al., 2015), respectively. The present study may be able to give a hint for why they failed to become planets. Additional constraints on their accretion histories are obtained from the mass distribution slopes of asteroids and KBOs.

Second, rapid migration of protoplanets caused by gravitational interactions with surrounding planetesimals is likely to occur once their masses reach $\sim M_{\rm oli}$. A protoplanet gravitationally scatters surrounding planetesimals. Back reaction on the protoplanet causes its radial migration, called planetesimal-driven migration (PDM), as the exerted torques do not cancel out (Bromley and Kenyon, 2011; Ida et al., 2000; Kirsh et al., 2009; Kominami et al., 2016;