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The Concentric Maclaurin Spheroid method with tides and a rotational enhancement of Saturn's tidal response

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ABSTRACT

We extend to three dimensions the Concentric Maclaurin Spheroid method for obtaining the self-consistent shape and gravitational field of a rotating liquid planet, to include a tidal potential from a satellite. We exhibit, for the first time, an important effect of the planetary rotation rate on tidal response of gas giants, whose shape is dominated by the centrifugal potential from rapid rotation. Simulations of planets with fast rotation rates like those of Jupiter and Saturn, exhibit significant changes in calculated tidal love numbers k_{nm} when compared with non-rotating bodies. A test model of Saturn fitted to observed zonal gravitational multipole harmonics yields $k_2 = 0.413$, consistent with a recent observational determination from *Cassini* astrometry data (Lainey et al., 2016.). The calculated love number is robust under reasonable assumptions of interior rotation rate, satellite parameters, and details of Saturn's interior structure. The method is benchmarked against several published test cases.

1. Introduction

The gas giants Jupiter and Saturn rotate so rapidly that adequate treatment of the non-spherical part of their gravitational potential requires either a very high-order perturbative, or better, an entirely non-perturbative approach (Hubbard et al., 2014; Hubbard, 2012, 2013; Wisdom, 1996; Wisdom and Hubbard, 2016). Here we present an extension of the Concentric Maclaurin Spheroid (CMS) method of Hubbard (2012, 2013) to three dimensions to include the tidal perturbation from a satellite. This allows for high-precision simulations of static tidal response, consistent with the planet's shape and interior mass distribution. The presence of a large rotational bulge produces an observable effect on the tidal response of giant planets. This effect, which has not been previously revealed by linear tidal-response theories applied to spherical-equivalent interior models, has implications for the observed tidal responses of Jupiter and Saturn.

The *Juno* spacecraft is expected to measure the strength of Jupiter's gravitational field to an unprecedented precision ($\sim$ one part in 10^9) (Kaspi et al., 2010), potentially revealing a weak signal from the planet's interior dynamics. Also present in Jupiter's gravitational field will be tesseral-harmonic terms produced by tides raised by the planet's large satellites. In fact, close to the planet, the gravitational signal from Jupiter's tides has a similar magnitude to the predicted signal from models of deep internal dynamics (Cao and Stevenson, 2015; Kaspi, 2013; Kaspi et al., 2010). An accurate prediction of the planet's hydrostatic tidal response will, therefore, be essential for interpreting the high-precision measurements provided by the *Juno* gravity science experiment.

Although the *Cassini* Saturn orbiter was not designed for direct measurement of high-order components of Saturn's gravitational field, it has already provided gravitational information relevant to the planet's interior structure. Lainey et al. (2016) used an astrometry dataset of the orbits of Saturn's co-orbital satellites to make the first determination of the planet's k_2 love number. Their observed k_2 was significantly larger than the theoretical prediction of Gavrilov and Zharkov (1977). A mismatch between an observed k_2 and the value predicted for a Saturn model fitted to the planet's low-degree zonal harmonics J_2 and J_4 would raise questions about the adequacy of the hydrostatic (non-dynamic) theory of tides.

In this paper we present theoretical results for simplified Saturn interior models matching the planet's observed low-degree zonal harmonics. When these models are analyzed with the full 3D CMS theory including rotation and tides, we predict a gravitational response in line with the observed k_2 value of Lainey et al. (2016), suggesting that the observation can be completely understood in terms of a static tidal response. A similar test will be possible for Jupiter once its k_2 has been measured by the *Juno* spacecraft.

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There is extensive literature on the problem of the shape and gravitational potential of a liquid planet in hydrostatic equilibrium, responding to its own rotation and to an external gravitational potential from a satellite; see, e.g., a century-old discussion in Jeans (2009). Many classical geophysical investigations use a perturbation approach, obtaining the planet's linear and higher-order response to small deviations of the potential from spherical symmetry. A good discussion of the application of perturbation theory to rotational response, the so-called theory of figures, is found in Zharkov and Trubitsyn (1978), while a pioneering calculation of the tidal response of giant planets is presented by Gavrilov and Zharkov (1977).

Hubbard (2012) introduced an iterative numerical method, based on the theory of figures, for calculating the self-consistent shape and gravitational field of a constant density, rotating fluid body to high precision. In the CMS method, integrals over the mass distribution are solved using Gaussian quadrature to obtain the gravitational multipole moments. This method was extended to non-constant density profiles by Hubbard (2013), by approximating the barotropic pressure-density relationship with multiple concentric constant-density (Maclaurin) spheroids. Here a spheroid is defined as a smooth shape obtained from deforming a sphere in three dimensions and is more general than an ellipsoid, whose shape is uniquely defined by 3 parameters. This approach mitigates problems with cancellation of terms that arise in a purely numerical solution to the general equation of hydrostatic equilibrium, and has a typical relative precision of $\sim 10^{-12}$. The CMS method has been benchmarked against analytical results for simple models (Hubbard et al., 2014) and against an independent, non-perturbative numerical method (Wisdom, 1996; Wisdom and Hubbard, 2016).

The theory of Gavrilov and Zharkov (1977) begins with an interior model of Saturn fitted to the values of J_2 and J_4 observed at that time. This interior model tabulates the mass density ρ as a function of s, where s is the mean radius of the constant-density surface. Tidal perturbation theory is then applied to this spherical-equivalent Saturn. The Gavrilov and Zharkov (1977) approach is sufficient for an initial estimate of the tidally-induced terms in the external potential, but it neglects terms which are of the order of the product of the tidal perturbation and the rotational perturbation. Here we demonstrate that, for a rapidly-rotating giant planet, the latter terms make a significant contribution to the love numbers k_{nm}, as well as (unobservably small) tidal contributions to the gravitational moments J_n.

Vorontsov et al. (1984) introduced a novel approach to calculation of the tidal response of giant planets. Rather than treating the problem as a purely static one, as we do here, they considered the case of a non-rotating giant planet orbited by a single satellite with an inertial orbital frequency Ω_s. They then calculated the response of the planet's normal oscillation modes to the perturbation, noting that the mode frequencies (whose oscillation periods are measured in hours) are much higher than satellite orbital frequencies (satellite periods are measured in days). For such off-resonance excitation, it is unnecessary to consider damping (as parameterized by the tidal quality factor Q) in calculating the tidal response. Taking the limit $\Omega_s \to 0$, Vorontsov et al. (1984) obtained the static tidal response of the non-rotating planet and thus its love number k_2. We compare the Vorontsov et al. Saturn k_2 with our value in Section 4.2, below.

An analogous problem has been studied for the tidal response of Galilean satellite Io by Zharkov (2004) and Zharkov and Gudkova (2010), and for close-in exoplanets by Correia and Rodríguez (2013). These works consider the second order approximations through a higher order perturbative theory. Our problem is different, however, in that the tidal and rotational perturbations for Io are of comparable magnitude, while the large influence of rotation on a much weaker tidal response found here for Saturn is unlike Io. Similarly, close-in, tidally locked exoplanets have comparable tidal and rotational perturbations.

Folonier et al. (2015) presented a method for approximating the love numbers of a non-homogeneous body using Clairaut theory for the equilibrium ellipsoidal figures. This results in an expression for the love number k_2 for a body composed of concentric ellipsoids, parameterized by their flattening parameters. In the case of the constant density spheroid, there is a well-known result that the equipotential surface is an ellipsoid. However, in bodies with more complicated density distributions, the equipotential surfaces will have a more general spheroidal shape. Because of the small magnitude of tidal perturbations, the method of Folonier et al. (2015) works in the limit of slow rotation despite this limitation. However, the method does not account for the coupled effect of tides and rotation, and does not predict love numbers of order higher than k_2. Within these constraints, we show below that our extended CMS method yields results that are in excellent agreement with results from Folonier et al. (2015).

Although our theory is quite general and can be used to calculate a rotating planet's static tidal response to multiple satellites located at arbitrary latitudes, longitudes, and radial distances, for application to Jupiter and Saturn it suffices to consider the effect of a single perturbing satellite sitting on an orbital plane at zero inclination to the planet's equator. Since tidal distortions are always very small compared with rotational distortion, and Jupiter's Galilean satellites, as well many of Saturn's larger satellites, are on orbits with low inclination, the tidal response to multiple satellites can be obtained by a linear superposition of the perturbation from each body. Extension of our theory to a system with a large satellite on an inclined orbit, such as Neptune-Triton, would be straightforward, but is not considered here.

2. Concentric Maclaurin Spheroid method with tides

2.1. Model parameters

In the co-rotating frame of the planet in hydrostatic equilibrium, the pressure P, the mass density ρ and the total effective potential U are related by

$$\nabla P = \rho \nabla U. \tag{1}$$

The total effective potential can be separated into three components,

$$U = V + Q + W, \tag{2}$$

where V is the gravitational potential arising from the mass distribution within the planet, Q is the centrifugal potential corresponding to a rotation frequency ω, and W is the tidal potential arising from a satellite with mass m_s at planet-centered coordinates (R, μ_s, ϕ_s), where R is the satellite's orbital distance from the origin, $\mu_s = \cos\theta$, where θ is the satellite's planet-centered colatitude and ϕ_s is the planet-centered longitude. In this investigation, we treat only the static tides in the co-rotating frame of the planet, and thus we always place the satellite at angular coordinates $\mu_s = 0$ and $\phi_s = 0$. The relative magnitudes of V, Q, and W can be described in terms of two non-dimensional numbers:

$$q_{\rm rot} = \frac{\omega^2 a^3}{GM} \tag{3}$$

for the rotational perturbation and

$$q_{\rm tid} = -\frac{3 m_s a^3}{M R^3} \tag{4}$$

for the tidal perturbation, where G is the universal gravitational constant, and M and a are the mass and maximum equatorial radius of the planet. The planet-satellite system is described by these two small parameters along with a third parameter, the ratio a/R.