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Two-dimensional Morlet wavelet transform and its application to wave recognition methodology of automatically extracting two-dimensional wave packets from lidar observations in Antarctica

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ABSTRACT

Waves in the atmosphere and ocean are inherently intermittent, with amplitudes, frequencies, or wavelengths varying in time and space. Most waves exhibit wave packet-like properties, propagate at oblique angles, and are often observed in two-dimensional (2-D) datasets. These features make the wavelet transforms, especially the 2-D wavelet approach, more appealing than the traditional windowed Fourier analysis, because the former allows adaptive time-frequency window width (i.e., automatically narrowing window size at high frequencies and widening at low frequencies), while the latter uses a fixed envelope function. This study establishes the mathematical formalism of modified 1-D and 2-D Morlet wavelet transforms, ensuring that the power of the wavelet transform in the frequency/wavenumber domain is equivalent to the mean power of its counterpart in the time/space domain. Consequently, the modified wavelet transforms eliminate the bias against high-frequency/small-scale waves in the conventional wavelet methods and many existing codes.

Based on the modified 2-D Morlet wavelet transform, we put forward a wave recognition methodology that automatically identifies and extracts 2-D quasi-monochromatic wave packets and then derives their wave properties including wave periods, wavelengths, phase speeds, and time/space spans. A step-by-step demonstration of this methodology is given on analyzing the lidar data taken during 28–30 June 2014 at McMurdo, Antarctica. The newly developed wave recognition methodology is then applied to two more lidar observations in May and July 2014, to analyze the recently discovered persistent gravity waves in Antarctica. The decomposed inertia-gravity wave characteristics are consistent with the conclusion in Chen et al. (2016a) that the 3–10 h waves are persistent and dominant, and exhibit lifetimes of multiple days. They have vertical wavelengths of 20–30 km, vertical phase speeds of 0.5–2 m/s, and horizontal wavelengths up to several thousands kilometers in the mesosphere and lower thermosphere (MLT). The variations in the extracted wave properties from different months in winter indicate a month-to-month variability in the gravity wave activities in the Antarctic MLT region.

1. Introduction

Observing and characterizing the field of atmospheric waves across various temporal and spatial scales has been, and continues to be, one of the most challenging tasks in atmospheric and space science research. Waves are the dominant mechanism for energy and momentum transport in the middle and upper atmosphere. Characterizing them is critically important to the understanding of circulation in Earth's atmosphere and ensuring the accuracy of numerical models that are used for climate prediction and weather forecasting. Advancements in remote sensing technologies in the last several decades have significantly improved the observing capabilities, providing volumes of data with unprecedented coverage, precision, and temporal and spatial resolutions. Efforts have been spent on analyzing the data to characterize the properties of various atmospheric waves, including gravity, tidal and planetary waves, by means of Fourier basis functions (e.g., Chu et al., 2011a; Forbes, 1995; Gardner and Voelz, 1987; Harris, 1994; Lu et al., 2015; Manson and Meek, 1986; Nakamura et al., 1993; Sato, 1994; She et al., 2004). However, because Fourier methods assume a time/space invariance of wave properties, more efforts are needed in the aspect of localization of these wave properties because, in reality, many waves are inherently intermittent and localized (Alexander and Dunkerton, 1999; Forbes et al., 1995; Sato and Yamada, 1994; Teitelbaum

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and Vial, 1991), i.e., wave properties such as amplitude, frequency, and vertical wavenumber vary with both time and space as the wave propagates. Traditional Fourier analysis is not capable of resolving localized variations (Farge, 1992). Windowed Fourier transforms are capable of localization, but use a fixed window width for all wave frequencies (or wavenumbers, same hereinafter) (Daubechies, 1992), which cannot dynamically adjust to lower or higher frequency phenomena. In contrast, the wavelet transform is able to adapt its window's temporal width to the wave spectrum, i.e., automatically narrowing at high frequencies and widening at low frequencies (Chui, 1992). Therefore, wavelet analysis methods are more suitable than Fourier transform methods for analyzing real atmospheric wave phenomena.

Wavelets are a relatively new concept in applied mathematics but have gained fast development and diverse applications. The term originated in the field of geophysics in the early 1980s (Morlet et al., 1982a, 1982b) to describe seismic signals. Since then, significant advances in wavelet theory have been made. Wavelets have been used in quantum physics (Grossmann and Morlet, 1984; Paul, 1984), applied mathematics (Daubechies, 1988; Meyer and Salinger, 1993), signal processing (Mallat, 1989), image compression (Wickerhauser, 1994), atmospheric turbulence (Farge, 1992), ocean wind waves (Liu, 1994) and many other fields. Their advantage, in addition to the localization capability, is the existence of many compactly supported orthonormal wavelet bases which allow signal decomposition into a minimal number of coefficients, enabling data compression (Daubechies, 1988; Meyer, 1989). The 1-D wavelet has been popular for middle and upper atmospheric data analysis (e.g., Sato and Yamada, 1994; Zhang et al., 2001; Zink and Vincent, 2001; Pancheva et al., 2002). Recent lidar observations in Antarctica, assisted with the 1-D Morlet wavelet data analysis technique, have led to the discoveries of persistent gravity waves in the mesosphere and lower thermosphere (MLT) (Chen et al., 2016a). These findings are significant, as they provide a rare insight into a poorly understood part of the Earth's atmosphere.

While the 1-D wavelet analysis applied in Chen et al. (2016a) was instrumental in characterizing the persistent waves from time series at each individual altitude, discerning each wave feature across both the time and space domains had to be performed manually; a situation that is not desirable when handling large amounts of observational data. In addition, wave packets may travel “obliquely” and exist in different regions at different times. Such intrinsic features require spectral analyses of waves in more than one dimension simultaneously. Many remote sensing instruments deliver two-dimensional (2-D) data. For example, ground-based lidars and radars record space-resolved atmospheric data over time at a fix location, yielding 2-D data in the altitude-time domains. Ground-based imagers, and many satellite sensors, obtain snapshots of 2-D spatial images. Unfortunately, extracting intermittent/localized two-dimensional wave packets is still a common technical challenge in analyzing atmospheric and space data. Therefore, we believe that the 2-D wavelet transform is an important and powerful tool for the autonomous processing of atmospheric data. Although some studies have used the 2-D wavelet transforms (e.g., Farge et al., 1990; Kaifler et al., 2015; Kumar, 1995; Wang and Lu, 2010), no 2-D Morlet wavelet code suitable for geophysical applications is publicly available. Moreover, the mathematical formalism is lacking regarding the absolute power spectrum of the 2-D wavelet transforms and the quantified relationship between the scale parameters and the Fourier periods/wavelengths.

The main goals of this study are to establish the mathematical formalism of modified 1-D and 2-D Morlet wavelet transforms with proper physical basis, and to develop a wave recognition methodology based on such 2-D wavelet transform for automatic wave extraction. The wave recognition methodology can be made available to the public for the analysis of large amounts of 2-D atmospheric and space science datasets. The formalism of the 1-D and 2-D wavelet transforms is established with detailed mathematical derivations. During the application of the 1-D wavelet technique in Chen et al. (2016a), problems were found in the publicly available 1-D wavelet code provided by Torrence and Compo (1998), i.e., wavelet power spectra are distorted or biased in favor of large scales or low frequencies, as Liu et al. (2007) pointed out previously for atmosphere and ocean science applications. To overcome this issue, corrections were made to the code in Chen et al. (2016a), but no mathematical explanations were offered. In this study we provide the mathematical and physical basis for the correction to the 1-D wavelet in Section 2 to illustrate the procedure for constructing an unbiased wavelet transform mathematically and to offer physical meaning to the wavelet spectrum. We then expand this procedure to the development of 2-D wavelet transform in Section 3. In order to apply the 1-D and 2-D transforms to real observational data, the continuous wavelet transform (CWT) must be discretized. This study provides equations for the discrete-time CWT in Sections 2 and 3. The 2-D inverse wavelet transform for wave reconstruction in the time/space domain is given in Section 4. Section 5 introduces the wave recognition methodology based on the modified 2-D wavelet transform that can automatically extract 2-D quasi-monochromatic wave packets and derive their wave properties. To demonstrate this methodology, we apply it to real lidar data taken during 28–30 June 2014 at McMurdo Station and compare our results to the 1-D results shown in Chen et al. (2016a). Application of the 2-D wavelet transform to lidar data from May and July 2014 is shown to help the characterization of persistent gravity waves in Antarctica. Section 6 discusses the potential caveats and improvements in our methods and the possible sources of these persistent waves. Finally, we conclude this study by highlighting the scientific utility of the 2-D wavelet transforms.

2. Correction for commonly used one-dimensional wavelet power spectrum

The 1-D CWT is commonly defined as (Mallat, 1999)

$$W_{f\psi}(s,\, t) = \int_{-\infty}^{+\infty} f(t')\frac{1}{\sqrt{s}}\psi^*\left(\frac{t'-t}{s}\right)dt' = f(t)\otimes\frac{1}{\sqrt{s}}\psi^*\left(\frac{-t}{s}\right) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\hat{f}(\omega)\sqrt{s}\,[\hat{\psi}(s\omega)]^* e^{i\omega t}d\omega \tag{1}$$

where $f(t)$ is the function of interest, $\psi(t)$ is a wavelet mother function, s is the wavelet scale (usually restricted to positive numbers), operator (*) denotes complex conjugate, operator ($\otimes$) denotes convolution, and operator ($\hat{\ }$) denotes Fourier transform, i.e., $\hat{f}(\omega) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega t}dt$ and $\hat{\psi}(\omega) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\omega t}dt$. The factor $1/\sqrt{s}$ is to ensure wavelets $\frac{1}{\sqrt{s}}\psi(\frac{t}{s})$ are normalized, i.e., $\int_{-\infty}^{+\infty}\left|\frac{1}{\sqrt{s}}\psi(\frac{t}{s})\right|^2 dt$ equals a constant and does not depend on s. Note that the third equality in Eq. (1) is a result of the convolution theorem, i.e., the Fourier transform of a convolution is the pointwise product of Fourier transforms, applying the inverse Fourier transform $f\otimes g = F^{-1}\{F(f)\cdot F(g)\}$. The wavelet scale s is proportional to the window width in a dilated wavelet $\psi(\frac{t'-t}{s})$. This property enables the wavelet “time-window” to dynamically adjust to higher or lower frequencies. The Morlet wavelet transform is provided to illustrate this feature and is defined as (Farge, 1992; Meyers et al., 1993; Weng and Lau, 1994),

$$\psi(t) = e^{i\omega_0 t}e^{-\frac{t^2}{2}}, \tag{2}$$