Journal of Atmospheric and Solar-Terrestrial Physics 161 (2017) 127–133

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar-Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

Study of auroral ionosphere using percolation theory and fractal geometry

A.A. Chernyshov[a,b,*], B.V. Kozelov[c], M.M. Mogilevsky[a]

[a] Space Research Institute of Russian Academy of Sciences, Profsoyuznaya 84/32, 117997, Moscow, Russia
[b] Moscow Institute of Physics and Technology, Institutskyi per. 9, 141700, Dolgoprudny, Moscow Region, Russia
[c] Polar Geophysical Institute of Russian Academy of Sciences, Akademgorodok 26a, 184209, Apatity, Murmansk Region, Russia

ARTICLE INFO

Keywords:
Auroral ionosphere
Nonlinear phenomena
Fractals
Percolation
Auroral phenomena

ABSTRACT

In this work, values of the fractal dimension and the connectivity index characterizing the structure of Hall conductivities on the night side of the auroral ionosphere are derived in general form. Restrictions imposed on fractal structure of the ionospheric conductivity are analyzed in terms of the percolation of the ionospheric Hall currents. It is shown that the structure of ionospheric Hall conductivities can be described as asymptotically path-connected fractal. This result is supported by analysis of typical structure observed in auroral electron precipitation which are also the main source of ionization on the night side of the ionosphere. It is demonstrated that crossing the precipitation region in the direction perpendicular to the multiple arcs system, one should observe the structure of the precipitation which looks like a generalized Cantor set.

1. Introduction

The understanding of fractal geometry of Nature can hardly be overrated. The term ”fractal” was introduced in science by Mandelbrot to quantify the geometric features of a variety of natural objects whose fine-scale structure is statistically self-similar (Mandelbrot, 1982). Unlike Euclidean geometry, he refused the implicit assumption about the smoothness of the object. Many object are in fact characterized by well-defined power-law spatial correlation function. In many cases, such power-law behavior could be associated with the fine-scale structuring in the system and the hierarchy of structures on many spatial scales could be then approximated by geometric sets termed fractals (Mandelbrot, 1982; Feder, 1988). Application of the fractal approach have led to considerable progress in many branches of science including problems of space physics, for instance, the study of processes on the Sun, solar wind, interplanetary magnetic field turbulence, stochastic substorm dynamics, Earth‘s distant magnetotail, the auroral structures and many others (for example, see (Zelenyi and Milovanov, 2004; Milovanov et al., 2001; Ohtani et al., 1995; Kozelov, 2003; Kozelov et al., 2004; Chang et al., 2010; Mogilevsky, 2001; Abel et al., 2009)).

Also, recently dynamic properties of the percolating networks near the critical threshold have received a good deal of attention. This insight along with the substantial advances in the geometric formulation of the critical phenomena have opened new perspectives on the topological methods in the theory of percolation (Stauffer and Aharony, 1994; Nakayama et al., 1994). This has led to an possibility of the geometric description of the dynamical phenomena involving the formation of the percolating structures. This approach has important advantages, because it allows one to consider a wider class of structures than has traditionally discussed. The geometric parameters of percolated clusters near the percolation threshold depend weakly on the details of the small-scale structure, which makes the percolation theory a promising tool for studying the properties of the medium. The relevance of the topological ideas applied to the auroral ionosphere has been recently demonstrated (Chernyshov et al., 2013b, a).

In the present paper, we use a geometric approach based on fractal theory and percolation theory to describe the Hall conductivity of the auroral zone ionosphere. Actually this study is a continuation of work initiated in the previous articles (Chernyshov et al., 2013b, a) where well-known in the literature empirical relations (Robinson et al., 1987; Spiro et al., 1982) were applied for determination of fractal parameters in auroral ionosphere and main attention was paid to the Pedersen conductivity. The obtained theoretical results for the Pedersen conductivity were in good agreement with electromagnetic field data from the satellites and ground-based observations of aurora. Using Spiro's relations (Chernyshov et al., 2013a) and Robinson's relations (Chernyshov et al., 2013b), different fractal results were determined for Hall conductivities. Therefore, it is necessary to find solutions in general form and to perform an analysis of obtained results. The Hall conductivity is important parameter in the auroral ionosphere because Hall current flows in auroral

* Corresponding author. Space Research Institute of Russian Academy of Sciences, Profsoyuznaya 84/32, 117997, Moscow, Russia.
E-mail addresses: achernyshov@iki.rssi.ru (A.A. Chernyshov), boris.kozelov@gmail.com (B.V. Kozelov), mogilevsky@romance.iki.rssi.ru (M.M. Mogilevsky).

http://dx.doi.org/10.1016/j.jastp.2017.06.013
Received 3 September 2015; Received in revised form 19 May 2017; Accepted 25 June 2017
Available online 27 June 2017
1364-6826/

arcs (Haerendel et al., 1996; Marghitu et al., 2011). This is the subject of this article.

The structure of the paper is the following. The next Section 2 is theoretical part of the article where expressions for the fractal dimension and the connectivity index for the Hall conductivity of the auroral ionosphere are specified. Comparison and confirmation of these theoretical estimations with experimental observations are carried out in Section 3. Discussion and concluding remarks are given in the last Section 4.

2. Fractal estimations of the ionospheric Hall conductivity

For the beginning we should remind the basic terms that are needed for following derivations. The introduction of the concept of fractals by Benoit B. Mandelbrot at the beginning of the 1970's represented a major revolution in various areas of physics (Mandelbrot, 1982). The problems posed by phenomena involving fractal structures may be very difficult, but the formulation and geometric understanding of these objects has been simplified considerably. Fractal structures are statistically self-similar. The statistically self-similar geometry appears in the power-law behavior of the average "mass" density of the fractal sets and this power-law behavior is contained in the factor a^{d_f-E}, here a is the length scale for the fractal sets and a is between two characteristic lengths; the microscopic distance and the correlation length (Mandelbrot, 1982). The parameter d_f in the is the so-called fractal dimension (or Hausdorff dimension) of the set and E is the dimensionality of the embedding Euclidean space, which is always not less than d_f. Note that in usual Euclidean geometry, the fractal dimension d_f coincides with the value of E so that the corresponding average density is constant. But the fractal dimension d_f is not the only geometric parameter required for the complete description of the self-similar fractals. The other important parameter is the index of connectivity θ. Index of connectivity describes the scaling behavior of the averaged "mass" density of a fractal set and θ quantifies how the elementary structural units inside the set are "glued" together to form the entire fractal object. In Euclidean geometry, the index θ is zero since d_f coincides with the Euclidean dimensionality E. The index of connectivity θ define the shape of a fractal object, and can be different for fractals even with equal values of the fractal dimension. Precise definition of θ could be given by using the concept of the geodesic line, that is, the shortest line connecting two elementary structural units of the fractal.

The E-region of nightside Earth's ionosphere at altitudes of 80– 150 km and at latitudes where the major part of energetic particles precipitation is observed and these particles result in auroras are considered. In this region, particle precipitation is the main cause of ionization in the nightside and, consequently, of increased conductivity. Using typical values of electron and ion gyro-frequencies and also the maximum values of the collision frequencies in E-layer of ionosphere, the expressions for the Hall conductivity in the E-region ionosphere are simplified and take the following form:

$$\sigma_H = \frac{qn}{B} \propto n \qquad (1)$$

where, n, q and B indicate electron density, electron charge and geomagnetic field strength. In the absence of other ionization sources the electron density is determined by ionization by auroral particles. The rate of ionization caused by auroral particles collisions with atmospheric gases varies smoothly along the magnetic field lines; therefore, the nontrivial fractal structure can form only in the spatial distribution transverse to the magnetic field.

By usual assumption of thin ionosphere we can go to height-integrated ionosphere (Swift, 1972) and to height-integrated conductivity $\sum_H = \int dz\sigma_H$, where z is the vertical coordinate. In the general case, the height-integrated Hall conductivity $\sum_H (W, \varepsilon)$ at given magnetic field line is a function of the average energy W and the energy flux ε of precipitating electrons. The dependence on the average energy is simply understandable because more energetic electrons penetrate deeper to the Hall current layer. According empirical models (Robinson et al., 1987; Spiro et al., 1982) this dependence has a power law form $\propto W^z$ with a bit different power index z. The dependence on energy flux has a form $\propto\sqrt{\varepsilon}$ due to recombination features at ionospheric E-region altitudes.

In important case of the region of intense field-aligned currents both W and ε depend on the current intensity. The field-aligned current $j_{||}$ generates transversal currents in the ionosphere and the corresponding transverse electric fields $E_\perp$, so we have $j_{||}\simeq\nabla_\perp(\Sigma\cdot E_\perp)$ (Wiltberger et al., 2009).

If a is the transverse characteristic scale, the change of potential drop in the ionosphere at this scale is $\Delta\varphi_\perp \sim j_{||}a_H^2/\Sigma_H$, and the change of field-aligned parallel potential drop is $\Delta\varphi_{||} \sim j_{||}$ (Lyons, 1981). So, we can obtain the following relation for the Hall conductivity:

$$\frac{\Delta\varphi_\perp}{\Delta\varphi_{||}} \propto \frac{a_H^2}{\Sigma_H} \qquad (2)$$

When the potential drop is much greater that the thermal energy of the source plasma, the energy flux ε is given by $\varepsilon = -e\Delta\varphi_{||}j_{||}$, and so, if the linear current-voltage relation holds, the energy flux is (Lyons et al., 1979; Lysak, 1990)

$$\varepsilon \propto \Delta\varphi_{||}^2 \qquad (3)$$

Note, that $W\simeq\Delta\varphi_{||}$, so, we have the reasons to consider the general case of empirical relation for the height-integrated Hall conductivity Σ_H in the following manner:

$$\Sigma_H \propto W^m \qquad (4)$$

here m is the free parameter, that is, scaling index in empirical approximation. It is assumed that $\sigma_H \sim \Sigma_H \propto W^m \propto (\Delta\varphi_{||})^m \propto (\Delta\varphi_\perp\Sigma_H/a_H^2)^m$ (see (Chernyshov et al., 2013b) for more details).

Now we need two additional facts about fractal sets and percolation. Firstly, the transverse potential drop is proportional to resistance $\Delta\varphi_\perp \propto R$, and the scaling of the resistance of fractal set with length is $R\propto a^\zeta$, where $\zeta = 2 + \theta - d_f$ is the resistance exponent (Havlin and Ben-Avraham, 1987), was derived earlier in the literature using arguments that the resistance on fractal set is changed significantly in comparison with the regular case. This is a consequence of the fact that in fractal structures holes of all sizes up to the size of the system exist. In other words, due to the presence of holes, bottlenecks and dangling ends in the fractal, the motion of a wandering particle is changed. Besides, since due to self-similarity, these holes, bottlenecks and dangling ends occur on all length scales, the motion of the wandering particle is changed on all length scales. Therefore, Fick's diffusion law is no longer valid and it is necessary to use a more general formula for the mean square displacement. The critical indexes of resistance and diffusion can be related by the Einstein equation. For this reason, it is obtained the estimation for resistance and critical resistance index (see (Bunde and Havlin, 2012)). Secondary, the criticality condition of the percolation threshold (the Alexander-Orbach conjecture) ensuing from the universal value theorem (Alexander and Orbach, 1982; Zelenyi and Milovanov, 2004) is

$$\frac{2d_f}{2+\theta} = \Lambda \approx \frac{4}{3} \qquad (5)$$

where, the parameter Λ characterizes the geometry of the percolation transition and determines the minimal fractional number of degrees of freedom that a particle must have to pass through a region under consideration in the process of random walks.

We will use this condition in form of inequality to estimate the values of fractal parameters that are necessary for the ionospheric current