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Ionospheric effects of magneto-acoustic-gravity waves: Dispersion relation

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ABSTRACT

There is extensive evidence for ionospheric effects associated with earthquake-related atmospheric disturbances. Although the existence of earthquake precursors is controversial, one suggested method of detecting possible earthquake precursors and tsunamis is by observing possible ionospheric effects of atmospheric waves generated by such events. To study magneto-acoustic-gravity waves in the atmosphere, we have derived a general dispersion relation including the effects of the Earth's magnetic field. This dispersion relation can be used in a general atmospheric ray tracing program to calculate the propagation of magneto-acoustic-gravity waves from the ground to the ionosphere. The presence of the Earth's magnetic field in the ionosphere can radically change the dispersion properties of the wave. The general dispersion relation obtained here reduces to the known dispersion relations for magnetoacoustic waves and acoustic-gravity waves in the corresponding particular cases. The work described here is the first step in achieving a generalized ray tracing program permitting propagation studies of magneto-acoustic-gravity waves.

1. Introduction

Hines (1972) first suggested that atmospheric gravity waves generated by tsunamis might produce identifiable ionospheric signatures that could be used for tsunami warnings, and Peltier and Hines (1976) concluded that such a system might be practical after determining that the various difficulties were of only marginal consequence. Similarly, there have been a variety of earthquake-related infrasonic signals documented by past researchers. For example, epicentral-generated infrasound measured at long ranges (e.g. Young and Greene, 1982; Mikumo, 1968) and infrasound measured by the local passage of Rayleigh waves (e.g. Bedard, 1971; Cook, 1965; Liu et al., 2011). Also, secondary radiation of infrasound from Rayleigh waves interacting with complex terrain has been measured (e.g. Young and Greene, 1982; Le Pichon et al., 2002).

The predictions of Hines (1972), and Peltier and Hines (1976) have been verified by observations taken of ionospheric effects of tsunami-generated atmospheric gravity waves during several recent major earthquakes (for example Artru et al., 2005; Hickey, 2011; Mai and Kiang, 2009; Liu et al., 2011; Makela et al., 2011).

Arai et al. (2011) have measured a Lamb wave radiated by a tsunami epicentral ocean surface disturbance. They suggest that by monitoring acoustic-gravity waves associated with undersea seismic disturbances it may be possible to indicate the likelihood of tsunami generation.

Other precursors have also been suggested (Varotsos et al., 1993, 2003; Freund, 2003; Geller, 1996). Finally, not only can infrasound be generated directly by a tsunami, Le Pichon et al. (2005) documented infrasound generated by the process of a tsunami interacting with a shoreline.

If it were possible to detect earthquake precursors soon enough to give warnings, lives could be saved. One suggested method of detecting earthquake precursors is by observing possible effects on the ionosphere of atmospheric waves generated by earthquake precursors (Blaunstein and Hayakawa, 2009; Heki, 2011), but that method is controversial (Masci and Thomas, 2015).[1] Testing the feasibility of such a warning system requires being able to calculate the propagation of such atmospheric waves from the ground to the ionosphere. Ray tracing programs exist for calculating the propagation of acoustic-gravity waves (e.g. Bedard and Jones, 2013; Jones and Bedard, 2015; Jones et al., 1986a, 1986b; Georges et al., 1990),[2] and estimates have been made for the propagation of acoustic/magnetoacoustic waves

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[1] Because seismic (Rayleigh) waves propagate much faster than sound, they are presently monitored in some locations as a precursor in early warning systems. There are also warning systems based on monitoring the positions of strategically chosen points in an earthquake zone using GPS technology (e.g. Heki, 2011). Here, we consider the possibility of monitoring the ionosphere as an alternative, additional warning system.

[2] There are also programs for calculating the propagation of acoustic waves in the atmosphere that are not ray based.

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from the ground to the ionosphere (Ostrovsky, 2008). However, as far as we know, no ray tracing program is now available to calculate the propagation of magneto-acoustic-gravity waves or even just magnetoacoustic waves in the atmosphere. Here, we derive the appropriate dispersion relations that could be used in a ray tracing program to make such calculations.

Estimating the ionospheric effects of atmospheric waves began at least by the 1960s (Georges, 1967; Yeh and Liu, 1972). Observation of atmospheric motions due to infrasound generation by earthquakes began as early as the 1960s. Due to the rapid decrease in gas density with altitude, the corresponding velocities and displacements can reach at least dozens of m/s and dozens of meters, respectively (Banister and Hereford, 1991; Pulinets, 2004; Krasnov et al., 2011; Rapoport et al., 2004; Heki, 2011, and the references therein). The role of magnetohydrodynamic effects in the evolution of infrasound entering the ionosphere from below had not been thoroughly studied until recently (Pokhotelov et al., 1995; Koshevaya et al., 2001; Ostrovsky, 2008). Ostrovsky (2008) analyzed the basic equations governing the propagation of sound from the ground to the ionosphere, and focused on understanding the main changes in the linear and nonlinear dynamics of an infrasonic wave propagating upward from the ground to ionospheric levels, where it transforms into the fast magnetic sound which is the same wave mode as the non-magnetic infrasound excited at lower altitudes. These calculations required some approximations, such as an exponential variation of density with height, a constant background magnetic field of the Earth, and making simple estimates for oblique propagation.

Here, we begin to extend the previous research by developing a general dispersion relation for magneto-acoustic-gravity waves, that could be used in an atmospheric ray tracing program to calculate the propagation of these waves from the ground up to the ionosphere. This will allow the calculations for arbitrary background models of temperature, density, pressure, winds, and the Earth's magnetic field, as well as extending the propagation to oblique propagation.

Hickey and Cole (1987) consider ionospheric mechanisms in more detail, including relative motion of ions and neutral molecules, as well as the role of viscosity and diffusion. Here we limit our approach to a simplified magnetohydrodynamic motion to apply to such sources as earthquake-generated magnetic sound.

Section 2 discusses how dispersion relations are used to construct WKB approximations following the method given by Weinberg (1962, Section IV). Section 3 gives the basic equations governing the propagation of magneto-acoustic-gravity waves. Section 4 linearizes the basic equations. Section 5 defines some of the notation.

Section 6 gives the dispersion relation for magneto-acoustic-gravity waves neglecting Coriolis force, vorticity, and rate-of-strain. This is later applied to examine wave properties for specific conditions.

Section 7 gives Hamilton's equations for the refraction and propagation of the rays that represent the waves determined by the system of coupled equations in Section 4. It is pointed out that the dispersion relation can be used for the Hamiltonian in Hamilton's equations in a ray tracing program even if the dispersion relation is given as the determinant of a matrix because Jacobi's formula can be used for the derivative of a determinant.

Section 8 discusses growth and decay of the waves because it is necessary when deriving a dispersion relation to distinguish between actual growth or decay and apparent growth of the waves when propagating to a region of low atmospheric density. We are reminded that baroclinicity causes growth or decay of waves because buoyancy is not a conservative force in a baroclinic fluid. However, growth or decay of a wave caused by baroclinicity must result in energy exchange between the wave and the mean flow if dissipation terms are neglected.

Section 9 considers the special case of a current-free region (that is, a region in which there are no background currents). Eq. (37) gives the magneto-acoustic-gravity-wave dispersion relation in a current-free region, which results in significant simplification. The resulting dispersion relation is used in further approximations to examine wave properties for specific conditions.

The barotropic approximation is often a good approximation for acoustic-gravity-wave propagation in the atmosphere. Section 10 applies the barotropic approximation to the dispersion relation, resulting in (38) for the more general case and (40) in a current-free region.

Section 11 investigates the properties of the barotropic approximation to the magneto-acoustic-gravity-wave dispersion relation. A key result is that the effect of the magnetic field increases with altitude as the Alfvén speed increases due to the decrease in atmospheric density with height.

Section 12 considers the special case of magnetoacoustic waves and shows exact agreement with the dispersion relation given in previous work (Ostrovsky, 2008). Section 13 considers Hamiltonian ray tracing of magnetoacoustic waves and shows that a quartic equation must be solved to give the magnitude of the wave vector to initialize the ray-path calculation when specifying the frequency and wave-normal direction.

Section 14 summarizes the main result, which is the derivation of the magneto-acoustic-gravity-wave dispersion relation, which is a generalization of the acoustic-gravity-wave dispersion relation to include a magnetic field, or the generalization of the magnetoacoustic-wave dispersion relation to include gravity.

Appendix A presents the linearized coupled equations in matrix form. The dissipation terms are neglected.

Appendix B gives the dispersion relation for magneto-acoustic-gravity waves in terms of the determinant of the matrix that represents the linearized coupled equations when the dissipation terms are neglected.

2. WKB approximations

Jones (1996) reviews the practical aspects of ray tracing, the WKB approximation, and the limits of geometrical optics to calculate wave propagation in the atmosphere. Although the WKB approximation was given its present name after 1926 (Wentzel, 1926; Kramers, 1926; Brillouin, 1926), the method was discovered earlier (Liouville, 1836, 1837a, 1837b; Rayleigh (John William Strutt), 1912; Jeffreys, 1923).

There are several possibilities for calculating a dispersion relation for the waves associated with a system of differential equations. Sometimes it is possible to eliminate all of the dependent variables but one to get a single differential equation for one dependent variable. Alternatively, it is possible to use for the dispersion relation the determinant of a matrix based on the system of equations (e.g. Weinberg, 1962, Section IV), which is what we shall do here.

In either case, it is necessary to replace differential operators by frequencies or wavenumbers to get a dispersion relation. Although the choice of method leads to slightly different dispersion relations (Einaudi and Hines, 1970), resulting in slightly different ray paths, the resulting WKB approximations differ from one another by less than the error in the WKB approximation. There may be some controversy about whether a dispersion relation is unique (Einaudi and Hines, 1970; Godin, 2015; Weinberg, 1962; Jones, 2006).

The linearized momentum Eq. (9) in Section 4 contains velocity shear terms that end up in the corresponding dispersion relation for the Eikonal method. Olbers (1981) reasons that in a WKB concept only the local fields are retained in the dispersion relation and gradients (such as shear terms) enter only the propagation and refraction equations. However, that restriction cannot apply when trying to construct approximate solutions to a differential equation that already contains gradient terms. He further reasons that keeping the shear terms in the dispersion relation would be inconsistent if those terms were smaller than some of the terms that are neglected in the WKB approximation. Although that reasoning is persuasive, a counter viewpoint is also persuasive. Namely, that to remove any of those shear