Journal of Atmospheric and Solar--Terrestrial Physics 152 (2017) 30–40

Contents lists available at ScienceDirect

Journal of Atmospheric and Solar–Terrestrial Physics

journal homepage: www.elsevier.com/locate/jastp

A method for detecting equidistant frequencies in the spectrum of a wideband signal

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ARTICLE INFO

Keywords:
Signal processing technique
Correlation functions
Eigenfrequencies

ABSTRACT

Examples are presented of using a signal processing technique that allows equidistant frequencies to be detected in broad-band oscillation spectra. This technique is based on analyzing the amplitude and phase correlation functions (APCF) of the oscillations. Equidistant frequencies can be detected in any broad-band spectrum based on the presence of periodic peaks related to such frequencies in APCF functions. An example of processing 1D resonator oscillations serves to show that the relationship between the eigenfrequencies in the spectrum and the APCF function peaks is similar to that between the optical grating slits and the interference line image on the screen. The proposed signal processing technique allows the difference between two adjacent frequencies of such a "grating" to be measured. The same analogy is true for a 2D resonator. In the latter case, two equidistant eigenfrequency gratings are shown to be present in the spectrum. Each grating corresponds to the eigenfrequencies of a 1D standing wave along each of the coordinates of a 2D resonator. The effect of small non-equidistance of the eigenfrequencies on the distortion and the location of the correlation function peaks is examined. The examples of processing two 1-h intervals of geomagnetic pulsation records are used to demonstrate the applicability of the APCF technique for real recorded magnetospheric oscillations.

1. Introduction

This study should be seen as a methodological supplement and development of Polyakov (2014), as well as to earlier papers by Polyakov and Potapov (2001) and Polyakov (2010) that formulated and developed a new, original technique to analyze the harmonic structure of oscillatory processes. As an application, the APCF technique has already been successfully employed in investigating the structure of standing seismic waves in the Earth's shells (Polyakov, 2010) and in determining the first harmonic frequency in various 1D standing MHD waves in the plasmasphere and at its boundary (Polyakov, 2014). The results of these applications show that the technique and its software may prove useful in obtaining additional information in research projects requiring spectrum analysis.

Let us examine a segment of almost monochromatic oscillations containing small random variations in amplitude and phase. In this case, each separate oscillation in the record differs slightly from the others in shape, amplitude, and period. According to Gudzenko (1961), such oscillations can be regarded as a periodically non-stationary random process for which an ergodic theorem generalization is valid. This means that, from the ensemble of individual observations in the record segment, we can derive one, average, oscillation that recurs periodically throughout the segment. The algorithms used to determine the mean oscillation, as well as the amplitude and phase fluctuations were suggested by Gudzenko (1962). These algorithms were applied to the new spectral analysis method by Polyakov (2010, 2014).

The end product of the APCF technique are the amplitude and phase correlation functions of oscillations rather than mean oscillations themselves. The APCFs are defined as the difference between the amplitudes and phases of quasi-monochromatic and mean oscillations. When calculating the correlation functions, we used ensemble averaging for individual oscillations. The APCF method employs these functions to analyze the structure of eigenfrequencies in a spectrum of the input signal.

Unlike the above-mentioned papers, this research relies on an updated APCF technique that enables processing, not only quasi-monochromatic, but also any broad-band signals. The updated analysis algorithm allows us to detect the eigenfrequencies of various resonators in signals of natural origin - such as seismic vibrations or geomagnetic disturbances - with much greater accuracy than previous methods. Major changes to the processing procedures are associated with the filtering of an input signal (see Section 2). The use of a selected spectral filter function allows us to isolate a quasi-monochromatic signal from any broadband spectrum. The frequency of this signal is determined by one of the parameters of the filter function. Post-processing procedures (determining the mean oscillation, amplitude and phase correlation

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http://dx.doi.org/10.1016/j.jastp.2016.11.007
Received 20 August 2015; Received in revised form 25 November 2016; Accepted 30 November 2016
Available online 01 December 2016
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functions) remain unchanged (see Polyakov (2010, 2014)).

Polyakov (2010, 2014) has shown that correlation functions obtained by the APCF method are periodic sequences of repetitive peaks for broadband oscillations in a resonator or waveguide. Those papers give a universal empirical formula that relates the interval between adjacent peaks to the frequency of a quasi-monochromatic signal or the difference between adjacent eigenfrequencies of a resonator. Using this formula, we conducted a harmonic analysis of the spectra of broadband signals. For detailed comments to this formula, see Section 2 (Formula (4)).

This paper is mainly a methodological study. It aims at determining some laws that are characteristic for the processing technique and the end product of the APCF program. Section 3 presents further arguments in favor of formula (4): an analogy with the optical diffraction grating. It also gives a more precise qualitative definition for the parameters in its right-hand side. Sections 3 and 4 examine the effect of a weak non-equidistance of eigenfrequencies for 1D and 2D-resonators.

An important problem is related to 2D standing waves. It was demonstrated in Polyakov (2010) and Polyakov (2014) that for a 2D resonator the interval between the APCF function peaks was defined by relation (4). The only difference from a 1D resonator is that the difference eigenfrequencies assume, not one, but two different values. Each value corresponds to the eigenfrequencies of a 1D standing wave along one of the coordinates of the 2D resonator. We cannot regard a 2D standing wave as a superposition of two separate 1D waves. The eigenfrequencies of such a wave are simultaneously determined by two harmonic numbers and must be very non-equidistant. How did the eigenfrequencies of two 1D waves find themselves in relation (4), which was obtained for a 2D standing wave? Section 4 resolves this problem.

Section 5 presents the results for real records of geomagnetic disturbances processed with an updated version of the APCF software.

Unlike Polyakov (2010) and Polyakov (2014), all the figures in this paper compare the processing technique results to the original signal spectrum. This has proved useful for revealing many peculiarities of the APCF functions. For example, Figs. 3 and 4 below demonstrate how non-equidistant eigenfrequencies in the spectrum lead to distortions in the correlation function peaks and to changes in the interval between two adjacent peaks.

2. Detection of harmonic structures in broad band signals

This paper uses updated software. The changes concern the procedures related to the input signal filtration. The filtered narrow-band signal is represented as an ideal sinusoid, whose amplitude and phase fluctuations are determined by the remaining spectral region.

Moreover, procedures are added to determine the presence of repetitive peaks on correlation functions. Here is a brief list of the basic steps involved in the processing. A more detailed description of the algorithms used in the main program for signal processing is given in Gudzenko (1962).

2.1. Filtering conversion

To start preprocessing, we convert the input signal into a complex-valued function whose imaginary part is equal to zero. We use the FFT procedure to obtain a Fourier function. The real and imaginary parts of this function are then multiplied by the filter spectrum function $F(f)$. Upon an inverse Fourier transformation, we will consider the real part to be the filtered signal. The frequency dependence of the filter function is shown in Fig. 1. The narrow rectangle at frequency f_0, its width equal to one step of the discrete Fourier function, allows us to identify monochromatic oscillations in any wideband signal. The rest of the spectrum in the Δf band (provided $a = 0.01$) is converted into small fluctuations, which make these oscillations deviate from their ideal shape. As a result a quasi-monochromatic signal is produced.

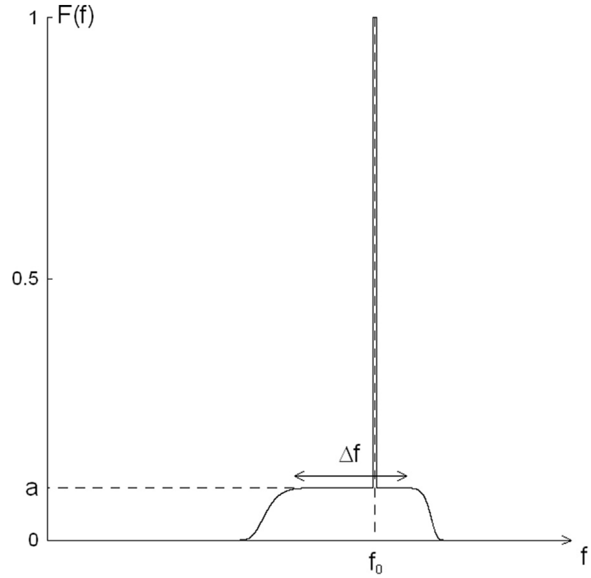

Fig. 1. Filter spectral function.

Therefore, the procedure of determining the mean oscillation is also applicable for it.

The position of the f_0 frequency on the horizontal axis can vary within the Δf band. An advantage of this approach is that it enables the parameters f_0, Δf and a to be varied at will. Fig. 1 presents a possible variant of their mutual location. For our processing technique, it is much more efficient than Marmet's filter (Marmet, 1979) used earlier in Polyakov (2010) and Polyakov (2014). Section 3 below shows how these parameters affect the final processing results.

2.2. Mean oscillation, amplitude and phase fluctuations

Let us address the time segment containing 100 periods $T_0 = 1/f_0$. This time segment moves in increments of $20T_0$ from the start to the end of the original signal time. All the subsequent processing procedures are applied to the filtered oscillations that are in this time segment after each increment.

The discrete values of deviation from zero x_i and time t_i become dimensionless after the conversion: $x_i \to x_i/\overline{A}$; $t_i \to 2\pi f_0 t_i$, where $\overline{A}$ is the mean oscillation amplitude. Next, the derivative $y_i = dx_i/dt$ is computed, and the points (x_i, y_i) are superimposed on the surface of the rectangular phase coordinates x , $\dot{x}$. Each oscillation, in these coordinates, is a closed trajectory (cycle) that differs little from a circumference of unity radius. In total, 100 cycles are obtained. Each cycle differs slightly from the rest. The Gudzenko technique (1962) is used to find the mean cycle that corresponds to the mean oscillation.

For each point (x_i, y_i) of the initial cycles, we determine the phase of the mean oscillation Θ_i and the deviation along the normal direction from the mean cycle $n(\Theta_i)$. The tangential deviations of the point are $\gamma(\Theta_i) = \Theta_i - t_i$. The normal and the tangential deviations n and γ represent the deviations of the oscillation amplitude and phase x_i from the amplitude and the phase of the mean oscillation.

It should be emphasized that the filtered oscillations at f_0 only serve as a "basis" to determine the amplitude and phase fluctuations. These oscillations per se do not take any part in any further procedures. The next stage deals only with fluctuations that do not contain frequency f_0 in their spectrum.

2.3. Correlation functions

For the discrete series $n(\Theta_i)$, $\gamma(\Theta_i)$ and for the derivatives $\frac{dn}{d\Theta}(\Theta_i)$, $\frac{d\gamma}{d\Theta}(\Theta_i)$, the cross- and auto-correlation functions are computed using

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