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Shapes of cometary isophotes with Maxwellian distribution of initial velocities for neutral molecules

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H I G H L I G H T S

- Modification of Bessel's equations for the axial cometary syndyne are presented.
- Accurate values of molecular acceleration in a cometary tail are given.
- The isophotes of comet C/1955 O1 (Honda) are described.

A R T I C L E I N F O

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This paper contains necessary modification of Bessel's equations for the axial cometary syndyne. This correction provides the accurate values of molecular acceleration in a cometary tail and precise values of decay constants for radiating molecules and their lifetimes. In consequence the hypothesis of the predissociation of molecules seems to be useless.

1. Introduction

Eddington (1910) explained by means of the Newtonian mechanics the sharp edge of a cometary tail. He assumed that molecules are emitted from the cometary nucleus by only single value of velocity to all directions. Molecules create spherical planes which radii are linearly growing in time of flight from the nucleus. Near the edge of tail the visible density is greater than inside of it. The whole tail is restricted by the edge of paraboloid (in the homogenous field of forces). Mokhnatch (1956) proved by means of analytical integration along a visual ray, that the visible density is in this case inversely proportional to the distance from the nucleus. Envelope of all molecules in the tail is that paraboloid, see above. If we replace a single value of emission velocity by the Maxwellian distribution of velocities we obtain the same results as Dolginov et al. (1971), (hereafter D71) for homogeneous field of forces. If we use the Maxwellian distribution of initial velocities, the edge of tail will be fuzzy but its brightness declines very quickly, the fastest in direction to the Sun, the most slowly to the opposite direction.

The spray-fountain model for emission of neutral particles from the cometary nucleus is used to derive the shape of isophotes (both in the central and homogenious field of forces), or for the description of syndynes or synchrones, see Finson and Probstein (1968). The equations of syndynes were firstly derived by Bessel (1836). The theory of neutral cometary tail was later developed by Eddington (1910), Mokhnatch (1956), Walace and Miller (1958), Haser (1966), Gnedin et al. (1971) or D71. The dusty cometary model was developed by Finson and Probstein (1968) and later was used by Sekanina and Miller (1973) for description of the comet C/1969 Y1 (Bennet).

The inverse numerical model of dust-tails based on the Maxwellian velocity distribution was presented by Fulle (1992). Fulle et al. (1992) applied this model to the CCD images of comet C/1988 A1 (Liller) and showed that dust-tail models are sensitive only to the most probable dust ejection velocity.

2. Choice of cometary model

One of the models with the isotropic Maxwellian distribution of initial velocities for homogeneous field of forces is possible to modify - using the Bessel's equations for axial syndyne - to the central field of forces presented in D71. Using the anisotropy factor k_f is possible to create a more general model of this type and to obtain a better approximation of observed isophotes than in D71. In this paper is demonstrated that the molecules in coma were emitted anisotropically rather than by an accidental jet-stream as was assumed in D71.

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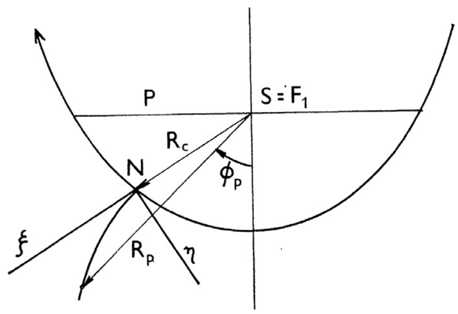

Fig. 1. The geometry of cometary axial syndyne.

The axial syndyne is created by centers of spherical planes, which are composed from the same molecules. They escape from the nucleus of comet by the same velocity in all directions and with the same acceleration, activated by the radiation pressure or by another forces, which have the origin in the Sun. From it follows, that centers of spherical planes have zero initial velocities relative to the nucleus. Radii of spherical planes grow with time of flight from the nucleus by linear manner. The outer envelope in the homogeneous field is created by paraboloids (see Eddington, 1910; Mokhnatch, 1956). They vary according to initial velocity and in the central field are deformed by the influence of motion of the cometary nucleus along the cone-section trajectory in gravity field of the Sun. Deformation of spherical molecular planes affected by gravity of cometary nucleus is practically neglected. If we want to correct numerical and graphical results, it is necessary to use - unlike Bessel - as a unit of time one year.

3. Correction of Besselian equations for axial syndyne

Bessel (1836) used as a unit of time so called *Bessel's Unit of Time*, $BUT = 1/k$ [days] $= 1/2\pi$ [years], where $k = 0.017202$ [$\mathrm{AU}^{3/2}/(\mathrm{M}_\odot{}^{1/2}$ day)], $\mathrm{M}_\odot$ is the mass of the Sun and k is Gaussian gravity constant ($k^2 = G$). From relation of molecular acceleration

$$a = G(1-\mu)\ \mathrm{M}_\odot/r^2 = 4\pi^2(1-\mu)/r^2$$

we can derive, that $1-\mu$ in the Besselian equation for cometocentric coordinates (3) of axial syndyne is necessary compensate by the term $(1-\mu)/(4\pi^2)$, because the time unit is 2π-times smaller and consequently the numerical value of time must be 2π-times greater. According to Finson and Probstein (1968) and Žáček and Vanýsek (1981) is clear that

$$\xi = r'\cos(v - v') - r, \tag{1}$$

$$\eta = r'\sin(v - v'),$$
$$r' = \frac{p}{\mu + (1-\mu)\cos(v' - v_1) + e\cos v'}, \tag{2}$$
$$r = \frac{p}{1 + e\cos v},$$

where $r'(v')$, $r(v)$, $r_1(v_1)$ are the length of radiusvector (resp. true anomaly) of particle on the axial syndyne, cometary nucleus and particle with nucleus together in the moment of separation particle from nucleus and p is the orbital element of a cometary nucleus. For the axial syndyne are all terms in the Besselian development of $\xi(\tau')$, $\eta(\tau')$ equal zero with the exception only one member for both coordinates (see Bessel, 1836)

$$\xi = \frac{(1-\mu)\tau'^2}{2r^2}, \eta = \frac{(1-\mu)2\sqrt{p}}{r^4}\frac{\tau'^3}{6}. \tag{3}$$

From here

$$\eta = \frac{\sqrt{p}(2\xi)^{3/2}}{3r\sqrt{1-\mu}}, \tag{4}$$

$$\tan\phi = \frac{2\sqrt{2p\xi}}{3r\sqrt{1-\mu}}, \tag{5}$$

where ϕ is angle of axial syndyne digression from the elongation of radiusvector Sun – cometary nucleus.

$$\cos\theta = \frac{r^2 + \Delta^2 - (1AU)^2}{2r\Delta}, \tag{6}$$

$$\xi' = \frac{\xi}{\sin\theta}, \tag{7}$$

where ξ' is the actual length of coordinate ξ. After change of the units then

$$\xi = \frac{k_b^2(1-\mu)}{r^2}\frac{\tau'^2}{2}, \eta = \frac{k_b^2(1-\mu)2\sqrt{p}}{r^4}\frac{\tau'^3}{6}, \tag{8}$$

where $k_b = 1/2\pi$ and

$$\eta = \frac{2\xi\sqrt{p(\xi + |\xi|)}}{3rk_b\sqrt{(1-\mu)\sin^3\theta}}, \tag{9}$$

$$\tan\phi = \frac{2\sqrt{p(\xi + |\xi|)}}{3rk_b\sqrt{(1-\mu)\sin\theta}}. \tag{10}$$

In Eqs. (9) and (10) the term 2ξ was replaced in numerator in square root by $(\xi + |\xi|)$, because the centers of spherical planes for $\xi < 0$ are not existed. Corrections of Eqs. (3)–(5) into the form (8)–(10) is necessary to obtain the correct numerical value also for tangent of angle of axial syndyne digression from the elongation of radiusvector Sun - cometary nucleus. If we don't change the unit *BUT* to year, the angle ϕ should be too small. For example for the comet C/1955 O1 Honda approx. 4°.9 instead of the measured value of 31° in the distance approximately $3.5 \cdot 10^5$ km from the cometary nucleus. The comet was from observer in a distance roughly 0.27 AU $= 4.04 \cdot 10^7$ km, which corresponds to visual angle approximately30'. If the angle ϕ had in these conditions (for original Bessel's equation without correction) reach 31°, the isophote should be on the side opposite to the Sun in a distance approximately $1.2 \cdot 10^7$ km from the cometary nucleus, which corresponds to the visual angle approx. 17°.5. The angular radius of cometary head was around20' only (Bacharev, 1955), which corresponds to dimension approx. $2.35 \cdot 10^5$ km in the direction perpendicular to the radiusvector, in evident contradiction with the value given above.

In opposite case, the acceleration should be approximately 40-times smaller than it has been derived for molecules C_2 by means of quantum mechanics in D71, namely $a = 3.95 \cdot 10^{-6}$ km/s^2, which corresponds to $(1-\mu) = 0.5762$ and $\mu = 0.4238$. In all cases we apply $4\pi^2 = 39.478$. If we assume this acceleration is correct, then also the Eqs. (8) – (10) of the original Bessel's Eqs. (3) – (5) for coordinates for axial syndyne and for angle of digression ϕ must be correct. Thus we can obtain an agreement of the observable data with the shape of isophotes of comet C/1955 O1 Honda on the sky.

From it follows the next consequence: the decay constant γ of radiating molecules must be 2π-times smaller and the lifetime of radiating molecules must be in the same ratio greater. The hypothesis of predissociation of molecules, which was supposed in some previous papers (Bouška and Vanýsek, 1967; Vanýsek and Žáček, 1967) to fulfil agreement between observations and theory is not necessary. Decreasing of the decay constant is necessary because