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The relativistic blackbody spectrum in inertial and non-inertial reference frames

Jeffrey S. Lee[a], Gerald B. Cleaver[a,b,*]

[a] Early Universe Cosmology and Strings Group, Center for Astrophysics, Space Physics, and Engineering Research, Baylor University, One Bear Place, Waco, TX 76706, US
[b] Department of Physics, Baylor University, One Bear Place, Waco, TX 76706, US

HIGHLIGHTS

- Semi-empirical derivation & applications of the blackbody spectrum.
- Inverse temperature deriving Relativistic Planck's & Wien's Displacement Laws.
- Motivation for inverse temperature applied to the blackbody spectrum.
- Relativistic spectral radiance.
- Equivalences between the observer and radiation frames.

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ABSTRACT

By invoking inverse temperature as a van Kampen-Israel future-directed timelike 4-vector, this paper derives the Relativistic Blackbody Spectrum, the Relativistic Wien's Displacement Law, and the Relativistic Stefan-Boltzmann Law in inertial and non-inertial reference frames.

1. Introduction

The semi-empirical derivation and applications of the blackbody spectrum for exclusively stationary radiation sources have been well-established and are in many physics textbooks. However, significant progress in establishing the relativistic blackbody spectrum has been stymied, at least to some extent, by unresolved issues in relativistic thermodynamics.

This paper makes use of the inverse temperature 4-vector to derive the Relativistic Planck's Law, the Relativistic Wien's Displacement Law, and the Relativistic Stefan-Boltzmann Law in inertial reference frames. In order to describe correctly the relativistic blackbody spectrum, *relativistic beaming* and *Doppler shifting*, in addition to *relativistic temperature transformation*, must be considered. Additionally, the non-inertial reference frame case is established with the azimuthally-constant 4-acceleration and, when necessary, the proper time derivative of the spectral radiance, the wavelength of maximum irradiance, and the radiation irradiance. Also, non-trivial solutions are sought for equal spectral radiances, equal wavelengths of maximum irradiance, and equal irradiances of the relativistic and radiation frame blackbody spectra.

The paper is organized as follows: motivation for inverse temperature being applied to the blackbody spectrum (Section 2), relativistic spectral radiance (Section 3), the relativistic Wien's Displacement Law (Section 4), the relativistic Stefan-Boltzmann Law (Section 5), and finally equivalences between the observer and radiation frames.

The primary motivation for this paper is the absence in the literature of a self-consistent treatment in establishing a relativistic version of Planck's Law leading to relativistic versions of Wien's Displacement Law and Stefan-Boltzmann Law. Ultimately, applications to the study relativistic plasmas, etc. may be realized.

* Corresponding author. Fax: +12547103878.
E-mail addresses: Jeff_Lee@Baylor.edu (J.S. Lee), Gerald_Cleaver@Baylor.edu (G.B. Cleaver).

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2. The application of inverse temperature to the blackbody spectrum

Although attempts have been made to develop the Relativistic Blackbody Spectrum (Johnson and Teller, 1982; Nakamura, 2009; Veitsman, 2013), these endeavors have been unsuccessful due, at least in part, to unresolved issues in relativistic thermodynamics (Lee and Cleaver, 2015; Przanowski, 2009; Einstein, 1907; Planck, 1908; Mosengeil, 1907; Pauli, 1921; Laue, 1921; Landsberg and Matsas, 1996). Disputes have arisen supporting three published Lorentz group transformations: Temperature Deflation (Einstein, 1907; Planck, 1908) and Temperature Inflation (Ott, 1963; Arzelies, 1965; Møller, 1967) (which can be operationally quantified with a relativistic Carnot cycle (van Laue, 1951; Tolman, 1987; Requardt, 2008)), and Temperature Invariance (Landsberg, 1966; Landsberg, 1967; Landsberg and John, 1970; Landsberg, 1981).

Significant misperceptions have arisen concerning temperature in relativistic thermodynamics due in part to the confusion surrounding the respective differences between empirical and absolute temperatures. The *empirical temperature* is a Lorentz invariant, relativistic scalar that considers the radiation rest frame and the observer frame to be in thermal equilibrium (Cubero et al., 2007). This ensues from the Zeroth Law of Thermodynamics, and correlates directly to the absolute temperature in the radiation (source) frame. The Zeroth Law's validity is required without making use of any thermodynamic property (including entropy and energy) (Kardar, 2007).

The *absolute temperature* of a thermodynamic system is a consequence of the Second Law of Thermodynamics. It is the product of the Lorentz factor and the absolute temperature in the radiation frame, and contains no angular dependence. Even though the difference between empirical and absolute temperatures may be observable in non-relativistic thermodynamics, it becomes persuasively illuminated in relativistic thermodynamics.

The Planck distribution describes a solid angular photon number density, and defines a *directional* (or *effective*) temperature. However, this results from solely mathematical manipulations, and its thermodynamic relevance is, at best, questionable. Alternatively, temperature transformations can be accomplished by treating *inverse temperature* as a van Kampen-Israel future-directed timelike 4-vector (Nakamura, 2009; Nakamura, 2005). Although Przanowski and Tosiek (Przanowski and Tosiek, 2011),[1] and Lee and Cleaver (Lee and Cleaver, 2015)[2] have demonstrated temperature inflation without making use of inverse temperature, angular dependence is required for the relativistic blackbody spectrum.

3. Derivation of the relativistic spectral radiance

The relativistic blackbody spectrum can be obtained by considering the blackbody spectrum of a stationary radiation source, and including temperature inflation (in terms of inverse temperature), Doppler shifting, and relativistic beaming. The inertial and non-inertial frames cases are each examined. In the non-inertial case, the Unruh Effect is not considered because it is many orders of magnitude smaller than the effect presented here.

3.1. Inertial frames

The radiation (source) frame photon energy density ε in frequency and wavelength spaces of a Planckian distribution are:

$$\varepsilon_\nu d\nu = \frac{\left(\frac{8\pi h\nu^3}{c^3}\right)}{\exp\left(\frac{h\nu}{k_B T_o}\right) - 1} d\nu \tag{1}$$

and

$$\varepsilon_\lambda d\lambda = \frac{\left(\frac{8\pi hc}{\lambda^5}\right)}{\exp\left(\frac{hc}{k_B \lambda T_o}\right) - 1} d\lambda \tag{2}$$

where h is Planck's constant, c is the speed of light, k_B is Boltzmann's constant, T_o is the rest frame absolute temperature, ν is the frequency, and λ is the wavelength.

Relativistically, the reciprocal of absolute temperature is replaced by the inverse temperature 4-vector[3]:

$$|\beta_\mu| = \left|\frac{u_\mu}{T_o}\right| = |\beta_t - \beta_z \cos\theta| \tag{3}$$

Also (de Parga et al., 2013):

$$\beta_t = \frac{1}{T_o\sqrt{1-V^2}} \tag{4}$$

$$\beta_z = \frac{V}{T_o\sqrt{1-V^2}} \tag{5}$$

T_o is the proper temperature (in the radiation frame).
u_μ is the relative 4-velocity between the radiation and the observer.
β_μ is the van Kampen-Israel inverse temperature 4-vector.
$V = \frac{u}{c}$ (fraction of light speed).
| | denotes magnitude of the vector quantity.

Although the azimuthally constant β_μ is the reciprocal of the effective temperature, given by Eq. (6), the inverse temperature arises directly from thermodynamic considerations, whereas the effective temperature is obtained entirely from mathematical manipulation. Although T_{eff} is adequate to determine planetary and stellar motion with respect to the CMB (Bracewell and Conklin, 1968; Henry et al., 1968; Peebles and Wilkinson, 1968), its thermodynamically non-physical origin leaves it unclear whether or not T_{eff} represents temperature. Additionally, relativistic beaming and Doppler shifting must be considered, and this is accomplished by introducing the Doppler factor $D = \sqrt{\frac{1+\beta}{1-\beta}}$.

$$T_{\text{eff}} = \frac{T_o\sqrt{1-V^2}}{1 - V\cos\theta} \tag{6}$$

Rewriting Eqs. (1) and (2) in terms of inverse temperature yields[4]:

$$\varepsilon'_\nu d\nu d\Omega = \frac{\left(\frac{8\pi h\nu^3}{c^3}\right)}{\exp\left[\frac{h\nu}{k_B}(\beta_t - \beta_z\cos\theta)\right] - 1} d\nu d\Omega \tag{7}$$

and

$$\varepsilon'_\lambda d\lambda d\Omega = \frac{\left(\frac{8\pi hc}{\lambda^5}\right)}{\exp\left[\frac{hc}{k_B\lambda}(\beta_t - \beta_z\cos\theta)\right] - 1} d\lambda d\Omega \tag{8}$$

From $B'_{\nu,\lambda} = \frac{c}{4\pi}\varepsilon'_{\nu,\lambda}$ and $\frac{B'_\lambda}{B_\lambda} = [\gamma(1 - V\cos\theta)]^{-3} = D^3$, Eqs. (7) and (8) become respectively:

$$B'_\nu d\nu d\Omega = \frac{\left(\frac{2h\nu^3}{c^2}\right)}{\exp\left[\frac{h\nu}{k_B}(\beta_t - \beta_z\cos\theta)\right] - 1} [\gamma(1 - V\cos\theta)]^{-3} d\nu d\Omega \tag{9}$$

and

$$B'_\lambda d\lambda d\Omega = \frac{\left(\frac{2hc^2}{\lambda^5}\right)}{\exp\left[\frac{hc}{k_B\lambda}(\beta_t - \beta_z\cos\theta)\right] - 1} [\gamma(1 - V\cos\theta)]^{-3} d\lambda d\Omega \tag{10}$$

[1] By using a superfluidity gedanken experiment.
[2] By rewriting the stress energy tensor using occupation number.
[3] In this paper, the case of the azimuthally constant inverse temperature vector is chosen (hence, $\beta_x = \beta_y = 0$), and consequently, all motion is along the z-axis.
[4] Primed quantities indicate the observer frame. Derivatives with respect to the proper time are denoted with dot notation.