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On the estimation of the current density in space plasmas: Multi-versus single-point techniques

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ABSTRACT

Thanks to multi-spacecraft mission, it has recently been possible to directly estimate the current density in space plasmas, by using magnetic field time series from four satellites flying in a quasi perfect tetrahedron configuration. The technique developed, commonly called "curlometer" permits a good estimation of the current density when the magnetic field time series vary linearly in space. This approximation is generally valid for small spacecraft separation. The recent space missions Cluster and Magnetospheric Multiscale (MMS) have provided high resolution measurements with inter-spacecraft separation up to 100 km and 10 km, respectively. The former scale corresponds to the proton gyroradius/ion skin depth in "typical" solar wind conditions, while the latter to sub-proton scale. However, some works have highlighted an underestimation of the current density via the curlometer technique with respect to the current computed directly from the velocity distribution functions, measured at sub-proton scales resolution with MMS. In this paper we explore the limit of the curlometer technique studying synthetic data sets associated to a cluster of four artificial satellites allowed to fly in a static turbulent field, spanning a wide range of relative separation. This study tries to address the relative importance of measuring plasma moments at very high resolution from a single spacecraft with respect to the multi-spacecraft missions in the current density evaluation.

1. Introduction

High resolution magnetic and plasma data in the interplanetary space have opened an important debate on the physical processes occurring between proton and electron scales. Indeed, spacecraft observations have revealed a steepening of the magnetic field power spectral density at scales at which the magnetohydrodynamics approximations are no longer valid, suggesting the presence of a small-scale turbulent cascade of magnetic energy (Leamon et al., 1998; Bale et al., 2005; Sahraoui et al., 2009, 2010; Alexandrova et al., 2008, 2009). This change of regime has been observed to occur at the proton gyroradius $\rho_p = v_{th,p}/\Omega_p$ ($v_{th,p}$ is the proton thermal speed and Ω_p the proton gyrofrequency) or at the proton skin depth $\lambda_p = c/\omega_p$ (c is the speed of light and ω_p the proton plasma frequency). In addition to the spectral properties, it has been found that the plasma is characterized by magnetic discontinuities at proton scales and sub-proton scales both in the pristine solar wind (Perri et al., 2012; Greco et al., 2016) and in the near Earth environment (Retinò et al., 2007; Sundkvist et al., 2007). These evidences have raised the question about the role played by magnetic field discontinuities and current-sheet like structures in the magnetic energy dissipation. The general picture emerging from the analysis of high frequency spacecraft data is the coexistence of oblique propagating Kinetic Alfvén Waves and zero-frequency coherent structures, namely current sheets-like structures (Roberts et al., 2015; Perschke et al., 2016). On the other hand, three dimensional numerical simulations have pointed out the emergence of current sheets over a broad range of scales (up to electron scales) as a consequence of the development of magnetic turbulence; these are sites of high concentration of current density, energy dissipation, and plasma heating (Karimabadi et al., 2013; Wan et al., 2015). One of the possible processes responsible for this local energy dissipation is magnetic reconnection, a change in the topology of the magnetic field leading to a conversion of magnetic energy into heat, particle acceleration, and non-thermal effects (Servidio et al., 2012; Valentini et al., 2014). Using high cadence measurements from the Magnetospheric Multiscale (MMS) mission, it has recently been possible to study in details electron scale magnetic reconnection and detect evidence of magnetic energy conversion into particle energy, electron currents, energy dissipation, and electron flows (Burch et al., 2016; Ergun et al., 2016; Yordanova et al., 2016; Fu et al., 2017). Owing to sub-proton inter-spacecraft

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separations (i.e., minimum average distance ~ 10 km), MMS offers the best condition for the estimation of the current density applying the *curlometer* method (Dunlop et al., 1988, 2002). This technique was already applied to Cluster data during periods of inter-spacecraft separation of ~ 200 km (Fu et al., 2012). Additionally, MMS measures three-dimensional plasma distributions with unprecedent time resolution, i.e., 150 ms for ions and 30 ms for electrons. Thus, high resolution current density can be derived from plasma moments as $\mathbf{J}_{mom} = qn(\mathbf{V}_i - \mathbf{V}_e)$, where q is the electric charge, $n = n_e = n_i$ is the plasma density, and $\mathbf{V}_i$, $\mathbf{V}_e$ are the ion and electron bulk speed, respectively (Graham et al., 2016). The consistency between the current computed from multi-spacecraft and derived from the moments is generally satisfactory except for regions where structures at scales below the spacecraft separation are present (Burch et al., 2016; Graham et al., 2016).

In this paper, in order to explore the limit of validity of the multi-spacecraft approach, we apply the curlometer technique to a synthetic model of stationary turbulence (Malara et al., In press; Pucci et al., 2016) where four virtual spacecraft are allowed to fly forming a perfect tetrahedron with adjustable inter-spacecraft separation. We discuss the implications for possible future single-spacecraft missions in the estimation of the current density, which is of pivotal relevance in the study of plasma turbulence and dissipation.

2. The curlometer technique

The curlometer technique has widely been used in recent years thanks to multi-spacecraft missions. It is based on the Maxwell-Ampère's law $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$ evaluated in the centre of a perfect tetrahedron formed by four satellites (see Fig. 1 in Dunlop et al. (1988)). Since this method is well known, here just a brief overview is given. Starting from the ideal aforementioned configuration, one can estimate the current density in the direction normal to each face of the tetrahedron. Under the assumption that the magnetic field does not change abruptly over the inter-spacecraft separation, that is it varies linearly and the current density is roughly constant over the entire volume of the tetrahedron, one can write (Grimald et al., 2012)

$$\mu_0 \mathbf{J}_{ijk} \cdot (\Delta \mathbf{r}_{ik} \times \Delta \mathbf{r}_{jk}) = \Delta \mathbf{B}_{ik} \cdot \Delta \mathbf{r}_{jk} - \Delta \mathbf{B}_{jk} \cdot \Delta \mathbf{r}_{ik}, \quad (1)$$

where i, j, k are index running over the satellites, so that $\mathbf{J}_{ijk}$ is the current density normal to the face delimited by spacecraft i, j, k. $\Delta \mathbf{B}_{ik} = \mathbf{B}_i - \mathbf{B}_k$ and $\Delta \mathbf{r}_{ik} = \mathbf{r}_i - \mathbf{r}_k$ are the magnetic field and the position difference between spacecraft i and k, respectively. Both the magnetic

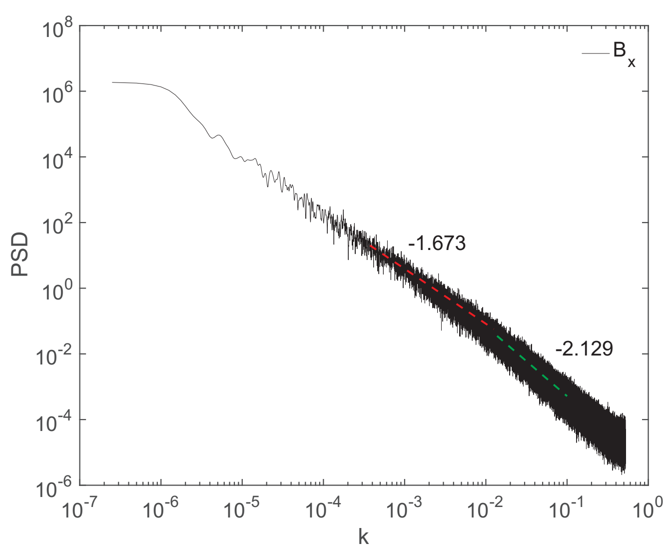

Fig. 1. Power spectral density of the B_x component as a function of scales. The two power law ranges are highlighted by the dashed lines.

field data and the spacecraft positions are in Cartesian coordinates and an average current density in the tetrahedron, $\mathbf{J}_{\mathrm{curl}}$, can be derived by projecting each current vector normal to three faces into Cartesian coordinates. Because of the assumption of slow variation of the magnetic field, it is clear that the curlometer can be applied only when spacecraft are close enough to avoid sudden variation in the field inside the tetrahedron volume. This tends to limit the goodness of the estimation of the current density via this method. To estimate the accuracy of the technique, one can evaluate $\nabla \cdot \mathbf{B}$, so that non-zero values are due to non-linear gradients in the magnetic field in the tetrahedron. Following, Dunlop et al. (1988), Grimald et al. (2012) we compute

$$div(\mathbf{B}) |\Delta \mathbf{r}_{ik} \cdot (\Delta \mathbf{r}_{jk} \times \Delta \mathbf{r}_{jl})| = |\sum_{cyclic} \Delta \mathbf{B}_{ik} \cdot (\Delta \mathbf{r}_{jk} \times \Delta \mathbf{r}_{jl})|, \quad (2)$$

and in particular we calculate the a dimensional quality factor

$$Q = \frac{div(\mathbf{B})}{\mu_0 \mathbf{J}_{\mathrm{curl}}}, \quad (3)$$

so that $Q \ll 1$ indicates very good estimation of the current density via the curlometer.

3. Application to synthetic data sets

3.1. Numerical setup

In order to test the limit of validity of the curlometer, we make use of a recently developed synthetic model of three-dimensional, static turbulence that reproduces the main characteristics of space plasma turbulence (Malara et al., In press; Pucci et al., 2016). The model mimics the turbulent cascade of energy from larger to smaller scales, and is based on an algorithm that allows to reproduce large spectral width and tunable spectral and intermittency properties, with very small computational requirements. A detailed description of the model can be found in Malara et al., In press, and a demonstration of its use can be found in (Pucci et al., 2016). The main concept of the model is to build the magnetic field $\mathbf{B}(\mathbf{r})$ at each point $\mathbf{r}$ of the domain as the superposition of scale-dependent magnetic field fluctuations, chosen as to reproduce the desired turbulence characteristics. This is done by introducing a hierarchy of cells at different spatial scales $\ell_m = \ell_0/2^m$. Here ℓ_0 is the domain size, $m = 0, \ldots, N_s$ is the scale index and N_s is the tunable number of scales used in the realization, defining the spectral extension. At the largest scale, there is a single cell of size ℓ_0. Then, each cell of a given scale is recursively divided into eight cells of the next scale size. At any given scale, the cells form a regular lattice filling the whole domain. Each cell in the model is indicated by four indexes $(i, j, k; m)$, where i, j, k identify the cell position within the 3D lattice at the m-th scale. For each cell, a spatially localized magnetic field fluctuation $\delta \mathbf{B}^{(i,j,k;m)}(\mathbf{r}, \ell_m)$, or magnetic eddy, is univocally assigned through suitably defined polynomial functions and a series of random numbers that control the energy "cascade". Such random numbers determine the amplitude of each eddy, in order to reproduce both a given global energy spectral law and the desired amount of fluctuations inhomogeneity (or intermittency). The amount of energy transferred from larger to smaller scales shall obey a given power-law spectrum $E(k) \sim k^{\alpha}$, which implies that, on average, the field fluctuations scale as $\delta B(\ell) \sim \ell^{(\alpha-1)/2}$. For example, a Kolmogorov spectrum with $\alpha = -5/3$ is obtained allowing a scaling $\delta B(\ell) \sim \ell^{1/3}$. Intermittency is modelled as in the standard p-model (Meneveau and Sreenivasan, 1987), where $p \in [0.5, 1]$ is the intermittency parameter. In this model, the energy flows from larger to smaller eddies with a rate proportional to p for some randomly selected cells, and to $1 - p$ for the remaining cells. For $p = 0.5$, the energy transfer rate is homogeneously redistributed in the cascade, resulting in the absence of intermittency. As p increases, the inhomogeneity of the energy transfer increases, enhancing the level of intermittency. Both the spectral index α and the intermittency level p