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Analysis for Cellinoid shape model in inverse process from lightcurves

Xiao-Ping Lu[a,d,*], Wing-Huen Ip[a,c], Xiang-Jie Huang[a], Hai-Bin Zhao[b,d]

[a] Macau University of Science and Technology, Avenida Wai Long, Taipa, Macau, China
[b] Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing, China
[c] Institute of Astronomy, National Central University, Taiwan
[d] Key Laboratory of Planetary Sciences, Chinese Academy of Sciences, Nanjing 210008, China

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ABSTRACT

Based on the special shape first introduced by Alberto Cellino, which consists of eight ellipsoidal octants with the constraint that adjacent octants must have two identical semi-axes, an efficient algorithm to derive the physical parameters, such as the rotational period, pole orientation, and overall shape from either lightcurves or sparse photometric data of asteroids, is developed by Lu et al. and named as 'Cellinoid' shape model. For thoroughly investigating the relationship between the morphology of the synthetic lightcurves generated by the Cellinoid shape and its six semi-axes as well as rotational period and pole, the numerical tests are implemented to compare the synthetic lightcurves generated by three Cellinoid models with different parameters in this article. Furthermore, from the synthetic lightcurves generated by two convex shape models of (6) Hebe and (4179) Toutatis, the inverse process based on Cellinoid shape model is applied to search the best-fit parameters. Especially, for better simulating the real observations, the synthetic lightcurves are generated under the orbit limit of the two asteroids. By comparing the results derived from synthetic lightcurves observed in one apparition and multiple apparitions, the performance of Cellinoid shape model is confirmed and the suggestions for observations are presented. Finally, the whole process is also applied to real observed lightcurves of (433) Eros and the derived results are consistent with the known results.

1. Introduction

Asteroids can provide the initial information about the formation of the solar system. The inverse process, to derive the physical parameters, including the rotational period, pole orientation, and the overall shape from the observed photometric data of asteroids, is one important field in the research on asteroids. Generally, there are two commonly used shape models with the corresponding algorithms based on the two models. First is the traditional triaxial ellipsoid model. Karttunen and Bowell (1989) introduced how to model the brightness variations of asteroids based on the ellipsoid shape and analysed the morphology of synthetic lightcurves. Cellino et al. (2009) presented the genetic inversion method to derive the physical parameters from the sparse photometric data based on the ellipsoid model. Carbognani et al. (2012) attempted to represent the asteroid shape by the ellipsoid for the Gaia photometric data. Muinonen et al. (2015) presented analytical formulas to derive the initial rotation, shape, and scattering properties for asteroids from sparse and dense photometry based on the so-called Lommel-Seeliger ellipsoid. Additionally, Cellino et al. (2015) applied the formulas in their genetic inversion method and confirmed the efficiency of the analytical formula based on triaxial ellipsoids. Second shape model is the convex polyhedron shape model. Kaasalainen et al. (1992) presented an inversion algorithm to derive this type shape model from multiple lightcurves observed in various viewing geometries. Moreover, they also introduced how to improve the computing speed by applying the Lebedev quadrature (Kaasalainen et al., 2012). Ďurech et al. (2010) applied the Kaasalainen's method to derive the shape models with the relevant physical parameters for about 900 asteroids.

As a mediate shape model between the ellipsoid and convex model, Cellinoid shape model was introduced first by Cellino et al. (1989), which consists of eight octants from ellipsoids. With asymmetric shape, Cellinoid shape model of having three more parameters than triaxial ellipsoid model, can simulate the shapes of real asteroids better. Compared with the convex shape model of having about 50 parameters, Cellinoid shape model can be applied in circumstances with the observed photometric data points less than 100. Based on the Cellinoid shape model, Lu et al. (2014) presented the whole inverse process to derive the physical parameters of asteroids from lightcurves. Furthermore, they also applied the inverse process to multiple light-

* Corresponding author at: Macau University of Science and Technology, Avenida Wai Long, Taipa, Macau, China.
E-mail address: xplu@must.edu.mo (X.-P. Lu).

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curves for refining the derived parameters and the sparse photometric data, such as Hipparcos data (Lu and Ip, 2015; Lu et al., 2016). The numerical experiments confirm the inverse process can derive the correct initial parameters and the applications to the observed lightcurves and Hipparcos data also show that it can derive the consistent results with the other methods, such as the genetic method (Cellino et al., 2009) and Kaasalainen's method (Ďurech et al., 2010).

For thoroughly investigating the relationship between the morphology of the synthetic lightcurves generated by Cellinoid shape model and the physical parameters, the numerical tests are implemented in this article. Besides, by applying the Cellinoid shape model to the synthetic lightcurves generated from two convex shape models at various viewing geometries, the scheme for searching the rotational period and pole of asteroids is presented.

The article is arranged as follows. In Section 2, the different lightcurves are shown and compared to the physical parameters. Subsequently, in Section 3, the two convex models for (6) Hebe and (4179) Toutatis are employed for generating the lightcurves. For better simulating the real observing circumstance, the orbits of the two asteroids are considered for guaranteeing the asteroids observable. Then the Cellinoid shape model is applied to search the best parameters. Finally, the scheme of efficiently applying the Cellinoid model to real observed lightcurves of (433) Eros is discussed in Section 4. The conclusion is to sum up in Section 5.

2. Morphology of synthetic lightcurves

2.1. Cellinoid shape model and brightness integration

As described in Lu et al. (2014), Cellinoid shape model is defined in terms of eight octants from eight similar ellipsoids with six semi-axes $\{a_1, a_2, b_1, b_2, c_1, c_2\}$. The upper four octants are listed as follows:

$$U1:\ \frac{x^2}{a_1^2} + \frac{y^2}{b_1^2} + \frac{z^2}{c_1^2} = 1, \quad x \ge 0, y \ge 0, z \ge 0 \tag{1}$$

$$U2:\ \frac{x^2}{a_2^2} + \frac{y^2}{b_1^2} + \frac{z^2}{c_1^2} = 1, \quad x \le 0, y \ge 0, z \ge 0 \tag{2}$$

$$U3:\ \frac{x^2}{a_2^2} + \frac{y^2}{b_2^2} + \frac{z^2}{c_1^2} = 1, \quad x \le 0, y \le 0, z \ge 0 \tag{3}$$

$$U4:\ \frac{x^2}{a_1^2} + \frac{y^2}{b_2^2} + \frac{z^2}{c_1^2} = 1, \quad x \ge 0, y \le 0, z \ge 0 \tag{4}$$

In addition to the pole orientation (λ, β) in ecliptic coordinate frame, rotational period (P) with the initial rotational phase angle (Φ_0), and the scattering function (S), the integrated brightness of Cellinoid shape model can be simulated as

$$L(E, E_0) = \iint_{C+} S(\mu, \mu_0, \alpha)\, d\sigma, \tag{5}$$

where E, E_0 are the viewing direction and illuminating direction respectively in ecliptic coordinate frame.

Here it should be noticed that as the Cellinoid shape model is asymmetric, the principal axis is not same as the one of ellipsoid model, i.e. z-axis. As shown in Lu et al. (2014), the principal axis of Cellinoid shape model with six semi-axes a_1, a_2, b_1, b_2, c_1, and c_2 can be calculated by the eigen decomposition of the inertia tensor matrix $(A - MB)$, where $M = \frac{\pi}{6}(a_1 + a_2)(b_1 + b_2)(c_1 + c_2)$ is the volume of Cellinoid, and

$$A = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix}, \tag{6}$$

is the inertia matrix constructed by nine components tensors, as well as

$$B = \begin{pmatrix} \bar{y}^2 + \bar{z}^2 & -\bar{x}\bar{y} & -\bar{x}\bar{z} \\ -\bar{x}\bar{y} & \bar{x}^2 + \bar{z}^2 & -\bar{y}\bar{z} \\ -\bar{x}\bar{z} & -\bar{y}\bar{z} & \bar{x}^2 + \bar{y}^2 \end{pmatrix}, \tag{7}$$

following the definition of the center of mass of Cellinoid shape model, $G(\bar{x}, \bar{y}, \bar{z}) = (\frac{3}{8}(a_1 - a_2), \frac{3}{8}(b_1 - b_2), \frac{3}{8}(c_1 - c_2))$.

Additionally, as described in Kaasalainen et al. (2001),

$$S(\mu, \mu_0, \alpha) = f(\alpha)[S_{LS}(\mu, \mu_0) + cS_L(\mu, \mu_0)] = f(\alpha)\left(\frac{\mu\mu_0}{\mu + \mu_0} + c\mu\mu_0\right). \tag{8}$$

the scattering function is applied in the simulation, with the definition of phase function,

$$f(\alpha) = a \exp\left(-\frac{\alpha}{d}\right) + k\alpha + b, \tag{9}$$

presented by Muinonen et al. (2009).

Following the previously described scattering function and shape model, the brightness integral (5) can be discretized as,

$$L(\omega_0, \omega) \approx \sum_{i=1}^{8}\left(\sum_{j=1}^{N}[S(\mu, \mu_0, \alpha)\Delta S_{i,j}]\right), \tag{10}$$

where i is the index of octants of Cellinoid, and j is the index of triangular facets of each octant, as well as facet area $\Delta S_{i,j}$.

2.2. Synthetic lightcurves

Supposing the longitude and latitude of the pole orientation is $(0°, 90°)$ in ecliptic frame and the rotational period is $P = 5$ h, three Cellinoid shape models with different semi-axes are given in Fig. 1, where six semi-axes of (A) are [5, 5, 4, 4, 3, 3] and [6, 4, 5, 3, 4, 2] for (B), as well as [7, 3, 5, 3, 3, 3] for (C). The X, Y, Z in Fig. 1 show the three geometric axes, while the X_0, Y_0, Z_0 show the principal inertia axes. The shape model will rotate around the principal axis Z_0 with the largest inertia moment. Apparently, the Cellinoid shape (A) is the same as a traditional ellipsoid with the three semi-axes [5, 4, 3], whose geometric axes are identical to the principal inertia axes. Contrastively, the Cellinoid shape (B) and (C) show the asymmetric shape structures merely with 3 more parameters than the ellipsoid.

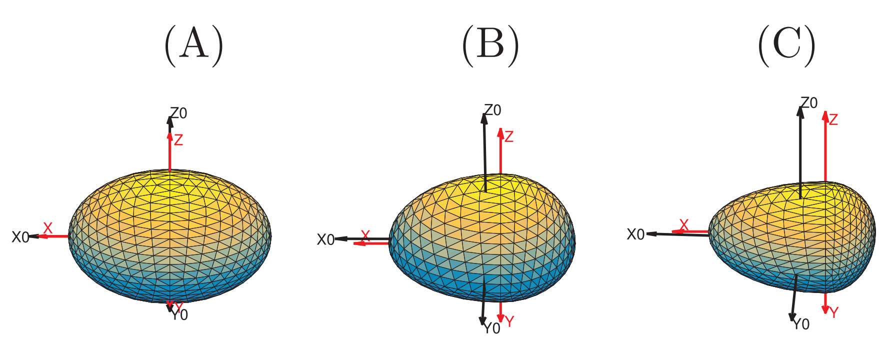

Fig. 1. Three Cellinoid shapes with different semi-axes. (X, Y, Z are the geometric axes; X_0, Y_0, Z_0 are the principal inertia axes).