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Stationary properties of a saturation laser stochastic system with time delay

Ping Zhu

School of Science and Technology, Puer University, Simao 665000, China

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ABSTRACT

Stationary properties of a saturation laser stochastic system with the time delay are investigated. Making use the method of the small time delay approximation, we obtain the analytic expressions of the stationary probability distribution $P_{st}(I)$ of the laser intensity, the nth power of the laser intensity $\langle I^n \rangle$, the normalized variance $\lambda_2(0)$, and the normalized skewness $\lambda_3(0)$ of the system. By numerical computation, we discuss effects of the delay time and the feedback strength on the stationary probability distribution $P_{st}(I)$, the mean value of the laser intensity $\langle I \rangle$, the normalized variance $\lambda_2(0)$ and the normalized skewness $\lambda_3(0)$. Above threshold the laser system possesses the efficient laser intensity output, and the delay time and the feedback strength enchances the laser intensity output of the saturation laser system. The delay time increases the stability of the laser intensity output. In contrast, the feedback strength weakens the stability.

1. Introduction

In the world of nature, many realistic systems can be regarded as to obey auto-regulating control, affect, and optimize mechanisms in terms of feedback loops by the output signal or other key quantities to the input, particularly, which can make the stochastic system driver by noise have the best and most effective output [1–9]. In the meanwhile, usually, the transmission takes some time, that is to say, the input signals are related to the output signals at earlier time, and the feedback effect of the output to the input possesses the time delay [10–16], including the laser system, the population system, the neural network system, the motor control system, and the chemical reaction system [17–28]. So for realistic stochastic systems, we should take into account the effects of time-delayed feedback. In recent years, many significant studying results on stochastic systems with linear time-delayed feedback and nonlinear time-delayed feedback have been obtained [29–48].

Statistical fluctuations in laser radiation determine the limits on the use of lasers in almost every application. The statistical properties of a saturation laser stochastic system that contains both additive and multiplicative noises are discussed by the experimental measurement and theoretical analysis [49–55]. Meanwhile, importance of the saturation effects on the behavior of the laser is shown [56–70].

Recently, the effects of delay-time feedback on the stochastic laser system have attracted the close attention of some researchers. Laser systems with optical delayed feedback are discussed both theoretically and experimentally [71–77]. Shiau et al. investigated a semiconductor microwave device with time-delay feedback, and obtained interesting results including the transition from monostability to multistability, complicated hysteresis loops, persistent bistability, and chaos [78].

E-mail address: zhuupp@163.com

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Gu et al. discussed the effects of the time-delayed feedback control in a single-mode laser stochastic system, pointing out the time-delayed feedback control can express the intensity fluctuation of the laser system [79]. The dynamical properties of the cubic model of a single-mode laser with two different types of time delays are shown in [80]. The saturation laser model is as typical as the cubic model of a single-mode laser model, which possesses the theoretical and applied values. However, in most of the previous studies of the stochastic system of the saturation model, the effects of time-delayed feedback are not considered further.

In this paper, time-delayed feedback in a saturation laser stochastic system is taken into account and effects of the time delay on the stationary properties of a saturation laser stochastic system are investigated. This paper is organized as followings: in Section 2, making use of the method of the small time delay approximation, we give the delayed Fokker–Plank equation (DFPE) for a saturation laser model driven by additive and multiplicative noises with time-delayed feedback, and derive the stationary probability distribution function of the laser intensity of the saturation laser stochastic system. Based on the numerical results, we discuss the effects of the time delay τ, the quantum noise intensity D, the pump noise intensity Q, and the feedback strength s on the stationary probability distribution, the mean, the normalized variance, and the normalized skewness of the steady-state laser intensity of the saturation stochastic system with the time-delayed feedback control in Section 3. Finally, summaries and conclusions of the results conclude the paper.

2. Theoretical analysis

The complex laser field E of a laser model with a full account of the saturation effects follows the Langevin Equation [56]

$$\frac{dE}{dt} = -kE + \frac{F_1 E}{1 + A \mid E \mid^2 / F_1} + \widetilde{p(t)}E + \widetilde{q(t)}, \tag{1}$$

where K is the cavity decay rate for the electric field and $F_1 = a_0 + K$ is the gain parameter; a_0 and A are real and stand for net gain and self-saturation coefficients. The random variables $\widetilde{q(t)}$ and $\widetilde{p(t)}$ are complex which present quantum noise and pump fluctuation, respectively. The statistical properties of the noise terms are characterized by their first and second moments:

$$\langle p(t) \rangle = \langle q(t) \rangle = 0, \tag{2}$$

$$\langle p(t)p(t') \rangle = 2Q\delta(t - t'), \tag{3}$$

$$\langle q(t)q(t') \rangle = 2D\delta(t - t'), \tag{4}$$

where D and Q stand for the strength of additive and multiplicative noise, respectively.

Performing the polar coordinate transform $E = re^{i\phi}$ on Eq. (1), one can obtain two equations of the field amplitudes r and phase ϕ. Then the Langevin equation of the field amplitude r can be written as follows [81]:

$$\dot{r} = -Kr + \frac{F_1 r}{1 + \frac{Ar^2}{F_1}} + \frac{D}{r} + rp(t) + q(t). \tag{5}$$

Considering the time-delayed feedback control in the laser system, Eq. (5) can be given as

$$\dot{r} = -Kr + \frac{F_1 r}{1 + \frac{Ar^2}{F_1}} + \frac{D}{r} + sr(t - \tau) + rp(t) + q(t), \tag{6}$$

where $s > 0$ represents the control strength and $\tau > 0$ denotes the delay time.

For the delayed Langevin equation of the field amplitude r including multiplicative and additive Gassian white noise terms, Eq. (6) can be transformed into the delayed equation including only an equivalent multiplicative white noise term as follows [82]

$$\frac{dr}{dt} = -Kr + \frac{F_1 r}{1 + \frac{Ar^2}{F_1}} + \frac{D}{r} + sr(t - \tau) + \sqrt{Qr^2 + D}\epsilon(t), \tag{7}$$

where

$$\langle \epsilon(t)\epsilon(t') \rangle = 2\delta(t - t'). \tag{8}$$

For a small time delay, we have [16]

$$r(t - \tau) = r(t) - \tau\frac{dr}{dt}. \tag{9}$$

For a large time delay, Eq. (9) should keep to higher powers of the time delay which can satisfy solution accuracy. Here we only consider a small time delay.