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Traveling wave solutions along microtubules and in the Zhiber–Shabat equation

E. Tala-Tebue[a,*], Z.I. Djoufack[a], D.C. Tsobgni-Fozap[b], A. Kenfack-Jiotsa[c], F. Kapche-Tagne[a], T.C. Kofané[b]

[a] Department of Telecommunication and Network Engineering, IUT-Fotso Victor of Bandjoun, University of Dschang, P. O. Box 134, Bandjoun, Cameroon
[b] Laboratory of Mechanics, Department of Physics, Faculty of Sciences, University of Yaounde I, P.O. Box 812, Yaoundé, Cameroon
[c] Nonlinear Physics and Complex Systems Group, Department of Physics, The Higher Teachers' Training College, University of Yaoundé I, P.O. Box 47 Yaoundé, Cameroon

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ABSTRACT

The aim of this paper is to apply the exponential rational function method for solving nonlinear equations arising in various physical models such as nonlinear electrical transmission lines, optical fibers, DNA, to mention a few. New exact solutions are obtained and can help to well understand the process of those systems. The method used is very effective and simple and can be applied to other types of nonlinear equations.

1. Introduction

The dynamics of many systems is usually described by nonlinear equations. To well understand the mechanisms of these physical models, it is important to solve the equations which characterize them. Growing interest has been focused on finding exact solutions of such systems. Up to now, there are many methods of building exact solutions in literature [1–16]. Nevertheless, not all equations proposed are solvable with the same method; that is why new methods for finding exact solutions for the governing equations of different systems have to be explored. In this work, based on the exponential rational function method, we derive many solutions of the equation of microtubules (MTs) [17] and the Zhiber–Shabat equation with other related equations [18].

The Zhiber–Shabat equation and related equations play an important role in different domains which are nonlinear optics, dislocation in crystals, kink dynamics, nonlinear optics, plasma physics, fluid dynamics and mathematical biology. Therefore, they deserve to have a particular attention. In Section 2, we present the method under consideration. This method will be applied in Section 3 and the summary of the work will be done in Section 4.

2. Method of finding solutions

In this section, we present the different steps of the exponential rational function method [19]:

* Corresponding author.
E-mail address: tebue2007@gmail.com (E. Tala-Tebue).

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1- Suppose that a nonlinear partial differential equation is given by

$$N\left(v, v^2, \ldots, \frac{dv}{dt}, \frac{d^2v}{dt^2}, \ldots, \frac{dv}{dx}, \frac{d^2v}{dx^2}, \ldots\right) = 0. \tag{1}$$

2- To solve this equation, we reduce the number of variables to only one. Thus,

$$v(x,t) = v(\xi). \tag{2}$$

And therefore, Eq. (1) constructs an ordinary differential equation (ODE) of the form

$$N(v, v^2, \ldots, v', v'', \ldots) = 0, \tag{3}$$

where $'$ denotes the derivation with respect to ξ. If it is possible, Eq. (3) can be integrated term by term one or more times.

3- According to the present method, a solution of Eq. (3) is expressed as follows:

$$v(\xi) = \sum_{i=0}^{m} \frac{a_i}{(1+e^{k\xi})^i}, \tag{4}$$

where k and a_i are unknown constants which will be determined. The parameter m is determined by balancing the linear terms of the highest order in the resulting equation with the highest order nonlinear terms. Substituting Eq. (4) into Eq. (3), we collect all coefficients of powers of $e^{k\xi}$ in the resulting equation where these coefficients have to vanish. This leads to a system of algebraic equations involving the parameters a_i and k. Solving this system with the aid of Maple, we obtain the exact solutions of Eq. (1).

3. Application of the method

3.1. Equation of MTs as nonlinear RLC transmission line

MTs are major architectural elements without which, the neuron could not achieve or maintain its exaggerated shape. They serve a transportation function, as they are the routes upon which organelles move through the cell. MTs are governed by the following equation [17]

$$R_2C_0l^2u_{xxt} + l^2u_{xx} + 2R_1C_0\delta uu_t - R_1C_0u_t = 0. \tag{5}$$

Setting

$$\xi = \frac{1}{l}x - c\frac{t}{\tau} \tag{6}$$

with $\tau = R_1C_0$, Eq. (5) leads to

$$u''' - \frac{\alpha}{c}u'' + 2\beta uu' - \gamma u' = 0. \tag{7}$$

It is important to notice that in (7), $\alpha = \frac{\tau}{R_2C_0}$, $\beta = \frac{\delta R_1}{R_2}$ and $\gamma = \frac{R_1}{R_2}$.

After integrating Eq. (7) regarding the constant of integration to zero, yields

$$u'' - \frac{\alpha}{c}u' + \beta u^2 - \gamma u = 0. \tag{8}$$

Balancing the highest order derivative term u'' and the nonlinear term u^2 in (8), we find $m = 2$. Thus, the solution has the following form:

$$u(\xi) = a_0 + \frac{a_1}{1+e^{k\xi}} + \frac{a_2}{(1+e^{k\xi})^2}. \tag{9}$$

Inserting Eq. (9) into Eq. (8), the constants c, k, a_0, a_1 and a_2 are obtained:

Case 1:

$$a_0 = a_2 = \frac{\gamma}{\beta}, a_1 = -\frac{2\gamma}{\beta}, c = -\frac{i\alpha}{5}\sqrt{\frac{6}{\gamma}}, k = i\frac{\sqrt{6\gamma}}{6}.$$

Therefore, we have:

$$u(x,t) = \frac{\gamma}{\beta}\left[1 - \frac{2}{1+\cos\frac{\sqrt{6\gamma}}{6}\xi + i\sin\frac{\sqrt{6\gamma}}{6}\xi} + \frac{1}{\left(1+\cos\frac{\sqrt{6\gamma}}{6}\xi + i\sin\frac{\sqrt{6\gamma}}{6}\xi\right)^2}\right]. \tag{10}$$

Case 2:

$$a_0 = a_2 = \frac{\gamma}{\beta}, a_1 = -\frac{2\gamma}{\beta}, c = \frac{i\alpha}{5}\sqrt{\frac{6}{\gamma}}, k = -i\frac{\sqrt{6\gamma}}{6}.$$