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Superconductivity in a SO(4) symmetric one-dimensional interacting system with diagonal three-body attraction

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ABSTRACT

Stimulated by recent experimental realization of tunability of multi-particle processes in ultracold fermion gases, we study a SO(4) symmetric generalized Hubbard model in an one-dimensional lattice with two-body and three-body interactions up to the nearest neighboring sites. By using the bosonization and renormalization-group schemes, we focus on weak-coupling regime and half-filled band, and determine the ground-state phase diagram, which consists of the Mott-insulator (MI), Luttinger liquid (LL) and Luther-Emery (LE) liquid phases. The diagonal three-body attraction significantly modifies the phase diagram and is responsible for superconducting phases even in the presence of two-body repulsions.

1. Introduction

The effect of interactions between electrons in the low-dimensional systems continues to be the current subject of intense investigation. Especially since the discovery of high-T_c superconductivity, much effort has been devoted to understanding electron correlation and superconductor transition. Both theoretical and experimental results exhibit that quite a number of strongly correlated electron systems demonstrate rich phase diagrams [1]. The main interest is focused on two-dimensional (2D) systems, but the study on one-dimensional (1D) systems is equally important. This is not only due to the conjecture [2] that properties of 1D and 2D counterparts of certain models share common aspects, but also due to the fact that the 1D case is easier to handle than its higher-dimensional versions. Moreover, there are some efficient theoretical schemes restricted to 1D systems, e.g., bosonization [3], “g-ology” renormalization group (RG) [4,5], and conformal field theory [6], which enormously facilitate investigation of 1D models as a first step. In addition, several numerical approaches are powerfully applied to 1D systems, such as quantum Monte Carlo [7], exact diagonalization [8] and density-matrix-renormalization group [9]. In addition to traditional quasi-1D Bechgaard salts [10], conducting polymers [11] and organic conductors [12], a great deal of 1D novel materials have been experimentally realized, such as carbon nanotubes [13], quantum wires [14], edge states in quantum Hall effect system [15]. All this highlights that the understanding of the 1D physics is both feasible and essential.

Usually, the correlation effects are appropriately modeled by the Hubbard Hamiltonian [16] and its generalizations [17–23]. These models are widely used to investigate various properties of 1D systems. Among others, the search for electronic superconductivity mechanism and the analysis of insulator-superconductor transitions are a topic of increased interest.

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Conceptually, the attractive Hubbard model is proposed to be the simplest model for describing the superconductivity. Analogous to negative-U Hubbard model, some phenomenological models with the BCS-like interactions were intensively studied [24,25]. Besides, the extended Hubbard model with correlated-hopping interactions (CHIs) provides a distinctive mechanism for superconducting instability [26], and the integrable supersymmetric extension of the Hubbard model with correlated kinematics is used to argue for a superconducting ground state of the η-pairing type [27]. However, in any case the electron Coulomb interactions are repulsive. Apparently, the mechanism of superconductivity is still an open question and is worth further clarifying.

In essence, the interacting electron systems belong to the many-body physics. In the context of nearest neighboring (nn) interactions, most of the Hubbard generalizations are restricted to two-body interactions. Even though the CHIs are considered, the interactions involve at most off-diagonal three-body form, which does not directly couple density–density interactions. However, the Pauli exclusion principle indicates the possible existence of diagonal three-body and even four-body interactions that directly couple local electron densities between intersites. These neglected many-body interactions may be more related to superconductivity or something else. Inspiringly, the experimental realization of ultracold fermion gases with strong dipolar moments and their confinement in optical lattices permit one to study in a controllable way effects of many-body diagonal interactions in 1D lattice systems [28–30]. For example, the diagonal three-body coupling has been experimentally observed in cold ^{85}Rb atoms confined in a magneto-optical trap [31] or in a optical lattice [32]. These experimental advances open a quite promising research frontier for investigation of extended Hubbard models, spurring the interest in the roles of three-body interaction and paving the way to the research for superconductivity. The present work is devoted to the investigation of such problem. Our main purpose is to explore the effect of recently experimentally controlled three-body site density coupling, which is expected to be relevant in stabilizing the superconducting phase. Particularly, for one value of the diagonal three-body attraction [which corresponds to the so(4) Lie algebra], we shall determine weak-coupling phase diagram, where in addition to an insulating spin-density-wave (SDW) phase, in appropriate regions of the parameter space the superconducting phase is the stable ground state.

2. Model and its low-energy analysis

The 1D interacting electron system that we consider is modeled by the full Hamiltonian:

$$H = H_1 + H_2 + H_3, \tag{1}$$

where

$$H_1 = -\sum_{i,\alpha} t(c^{\dagger}_{i\alpha}c_{i+1\alpha} + c^{\dagger}_{i+1\alpha}c_{i\alpha}), \tag{2}$$

$$H_2 = \frac{1}{2}\sum_{ijkl}\sum_{\alpha\alpha'}\langle ij|\mathcal{V}_{\alpha\alpha'}|kl\rangle c^{\dagger}_{i\alpha}c^{\dagger}_{j\alpha'}c_{l\alpha'}c_{k\alpha}, \tag{3}$$

$$H_3 = \frac{1}{2}\sum_{i\alpha} T(n_{i\alpha}n_{i\overline{\alpha}}n_{i+1} + n_{i+1\alpha}n_{i+1\overline{\alpha}}n_i). \tag{4}$$

Here, the H_1 term denotes the genuine hopping. The H_2 term represents two-body interactions. In the case of short-ranged electron interaction, the dominant matrix elements read $U = \langle ii|\mathcal{V}_{\alpha\overline{\alpha}}|ii\rangle$, $V = \langle ij|\mathcal{V}_{\alpha\alpha'}|ij\rangle$, $W = \langle ii|\mathcal{V}_{\alpha\alpha'}|jj\rangle$ and $X = \langle ii|\mathcal{V}_{\alpha\alpha'}|ij\rangle$. U and V parameterize the on-site and a nn density interaction, respectively. X is a bond-charge interaction, which may be viewed as a density-dependent nn hopping. W parameterizes the on-bond interaction, which is often referred to as a pair-hopping. Physically, $U, V, W, X > 0$. In addition, the U and V terms describe diagonal parts of the electron–electron interactions in a site representation, while the W and X terms describe site-off-diagonal parts. In Ref. [16], only the U term is considered in point of $3d$ electrons in transition metals. However, the neglected interactions cannot be expected to be irrelevant to the physics of the systems. When all the nn interactions are taken into account, the H_2 term is rewritten in the Wannier represent as

$$\begin{aligned} H_2 = {} & U\sum_i n_{i\uparrow}n_{i\downarrow} + V\sum_i n_i n_{i+1} \\ & + W\sum_i (c^{\dagger}_{i\uparrow}c^{\dagger}_{i\downarrow}c_{i+1\downarrow}c_{i+1\uparrow} + h.c.) \\ & + X\sum_{i,\alpha}(c^{\dagger}_{i\alpha}c_{i+1\alpha} + h.c.)(n_{i\overline{\alpha}} + n_{i+1\overline{\alpha}}). \end{aligned} \tag{5}$$

Distinctively, the H_3 term represents contribution of the three-electron processes. T is the diagonal three-body interaction, which directly couples local site densities, just like two-body U and V. Experimentally, these diagonal interactions (U, V, T) can be tuned independently by using the current ultracold systems [29,31–33]. Note that T itself is not equal to the Coulomb interaction but is a sign of the many-body effect, and hence it may be attractive ($T < 0$).

The model Hamiltonian (1) is fairly general, containing some interesting limiting cases. The known $t-U-V$ model corresponds to vanishing W, X and T, and its quantum phase diagrams were extensively discussed [34–41]. When V, X and