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Multiplicative non-Gaussian noise and additive Gaussian white noise induced transition in a piecewise nonlinear model

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ABSTRACT

We study the transition problems in a piecewise nonlinear model induced by correlated multiplicative non-Gaussian noise and additive Gaussian white noise. Firstly, applying the path integral approach, the unified colored noise approximation, the analytical expression of the steady-state probability density function (SPD) is derived. Then the change regulation of the SPD is analyzed with the change of the strength and relevance of multiplicative noise and additive noise. From numerical computations we obtain some new nonlinear phenomena: the transition can be induced by the cross-correlation strength between noises, the non-Gaussian noise intensity and the Gaussian noise intensity as well as the non-Gaussian noise deviation parameter. This indicates that the effect of the non-Gaussian noise intensity on SPD is the same as that of the Gaussian noise intensity. Moreover, we also find the correlation time of the non-Gaussian noise can not induce the transition.

1. Introduction

In the last few decades, as a non-linear phenomenon, noise induced transition has attracted considerable attention [1–10]. Noise-induced transition is an effect in which changing the noise intensity causes the transition to the new state, and in non-equilibrium systems it can be considered as a generalization of phase transitions in thermodynamic equilibrium systems. As an important characteristic value that is used to characterize the phenomenon of system noise-induced transition, steady-state probability density function has been investigated deeply by many researchers in fields now [1–20]. Horsthemke and Malek-Mansour [3] studied the influence of external noise on non-equilibrium phase transitions. Arnold et al. [4] studied the white and colored external noise and transition phenomena in nonlinear system. Van den Broeck and collaborators [5,6] proposed two general spatially extended models that describe the effect of multiplicative noise, and reported that the models can undergo a non-equilibrium phase transition leading to a symmetry-breaking state. Castro et al. [7] analyzed a model that has been shown to undergo a purely noise induced, from a monostable regime to a bistable one, when it is submitted to a colored noise source. Mangioni et al. [8,9] investigated the disordering effects of color in non-equilibrium phase transitions induced by multiplicative noise. Petrosyan and Hu et al. [10–14] studied the noise-induced phase transitions in some real world systems. As we known, noise always has internal (additive noise) and external (multiplicative noise) origins. The effect of interference of additive noise and multiplicative noise when these two kinds of noises are

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correlated has been researched in Refs. [15–23]. For example, Li and Huang [15] studied the non-equilibrium phase transition in a spatially extended model driven by the correlated noises. Jia et al. [16] investigated the SPD of a bistable system subjected to the additive and multiplicative noises with a colored cross-correlation. Through applying the unified colored-noise approximation, Cao and Wu et al. [17–19] obtained the expression of the SPD of the bistable kinetic model and single-mode laser subjected to correlated noises, and analyzed their statistical properties. Yang and Wang [20] investigated the steady state characteristics of the one-dimensional FitzHugh-Nagumo neural system with two different kinds of colored noises.

At present, in the study of noise-induced transition and its related problems, Gaussian noise and non-Gaussian noise have been successfully researched in aspects of theory and experiment. However, the research theory of non-Gaussian noise is not as mature as Gaussian noise. In fact, non-Gaussian noise can not be ignored, and it plays an irreplaceable role in the system. Some recent researches show that the non-Gaussian noise often appears in biological systems, physical systems and neural systems, etc. [24–34]. Among them, the area of non Gaussian noises did a notable advance as shown in Ref. [26]. The stochastic resonance(SR) and noise-induced transition was investigated by Fuentes and Wio et al. [27–29] when the system is driven by a noise source taken as colored and non-Gaussian. Wu et al. [30,31] analyzed the phase transition and the SR of a bistable system with coupling between non-Gaussian and Gaussian noise terms. Bag and Hu et al. [32,33] studied Kuramoto model of globally coupled phase oscillators subject to Ornstein-Uhlenbeck and non-Gaussian colored noise. Xu et al. [34] investigated the phase transitions and the mean first passage time of an asymmetric bistable system driven by non-Gaussian Levy noise.

However, above these studies were performed mainly for continuous system driven by Gaussian or non-Gaussian noise. The study about the piecewise nonlinear system is still relatively rare. For many experimental systems, the models are based on piecewise system [35–41]. Such as, electronic circuits, controllers and superconducting devices etc. Simpson et al. [35] studied the mixed-mode oscillations in a stochastic, piecewise-linear system. Fiasconaro and Spagnolo [36] studied the resonant activation in piecewise linear asymmetric potentials. Wang et al. [37] investigated SR in a bistable sawtooth system driven by correlated multiplicative and additive Gaussian white noises. Liang et al [38,39] propose a new parabolic-bistable potential model with an additive Gaussian color noise source, and studied the SPD, the mean first-passage time and the phenomenon of SR in this system. Jin and Li [40] investigated the mean first-passage time in piecewise nonlinear system driven by multiplicative and additive Gaussian white noises with colored cross-correlation. But so far there has been little research on the theory of the effects of non-Gaussian noise on piecewise nonlinear system [41]. This provides motivation to investigate the steady-state problems of a piecewise nonlinear model driven by correlated multiplicative non-Gaussian noise and additive Gaussian white noise.

In this paper, we study the transition problems in a piecewise nonlinear system induced by the non-Gaussian and Gaussian noises. In Section 2, the model of a piecewise nonlinear bistable system is introduced. In Section 3, we obtain the FPK equation of system and the exact expression of SPD with the unified colored noise approximation. In Section 4, the SPD is presented and the influences of the non-Gaussian noise intensity, the non-Gaussian noise deviation parameter, the Gaussian noise intensity, the correlation time of the non-Gaussian noise and the cross-correlation strength between noises on SPD are discussed. And some conclusions are drawn in Section 5.

2. The piecewise nonlinear system

Considering a piecewise nonlinear system driven by correlated multiplicative non-Gaussian noise and additive Gaussian white noise, the stochastic dynamics is subjected to the following Langevin equation:

$$\frac{dx}{dt} = -U'(x)+x(t)\eta(t)+\xi(t), \tag{1}$$

The deterministic part of the system (1) is piecewise function, which corresponds to the potential function $U(x)$ [35–41]

$$U(x) = \begin{cases} \frac{a}{2}(x+1)^2 + \frac{k}{2}, x < -\frac{\sqrt{3}}{3}, \\ \frac{b}{2}x^2, \qquad |x| \le \frac{\sqrt{3}}{3}, \\ \frac{a}{2}(x-1)^2 + \frac{k}{2}, x > \frac{\sqrt{3}}{3}. \end{cases} \tag{2}$$

and the parameter $a > 0$, $b < 0$, k is constant, $U(x)$ has two stable states $x_{s1} = -1$, $x_{s2} = 1$ and an unstable state $x_u = 0$ The potential function $U(x)$ image shown in Fig. 1.

The parameters are chosen as follows: ($a = 0.102567$, $b = -0.0866025$, $k = -0.0472$).

In Eq. (1), the noise term $\eta(t)$ has a non-Gaussian distribution with [26–31]

$$\frac{d\eta(t)}{dt} = -\frac{1}{\tau}\frac{d}{d\eta}V_q(\eta) + \frac{1}{\tau}\varepsilon(t), \tag{3}$$

and

$$V_q(\eta) = \frac{D}{\tau(q-1)}\ln\left[1 + \frac{\tau(q-1)}{D}\frac{\eta^2}{2}\right], \tag{4a}$$