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A study on proton-deuteron elastic scattering at intermediate energies

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ABSTRACT

Proton-deuteron elastic scattering differential cross section at intermediate energies is calculated. Including the three-nucleon force and backward scattering in the scattering amplitude the results of optical limit approximation for proton-deuteron elastic scattering differential cross section are improved. However, neglecting the double scattering terms in the optical limit approximation leads to the deviation of the results from the experimental data at minimum region where the double scattering has an effective role.

1. Introduction

Glauber multiple scattering theory [1] has been used before to study the effect of three-nucleon force on $p-{}^3He$ (${}^3H$)elastic scattering differential cross sections at high energy [2–4]. The same theory and optical limit approximation (OLA) of Glauber [1] were also used to study the same effect in hadron-deuteron $(h-d)$ elastic scattering at high energy [5,6]. The results of these studies show that the inclusion of three-nucleon force in elastic scattering amplitude is important and leads to improvement of the results. However, the multiple scattering theory and the OLA, in general, are not used to study these reactions at intermediate energies. The chiral perturbation theory [7,8] with the next-to-next-to-leading order and exact solutions of three-nucleon Faddeev equations [9–13] with three-nucleon force potentials, for example: modified Tucson-Melbourne and Urbana-IX three-nucleon force potentials [9], were extensively used to study three-nucleon force effect on $p-d$ elastic scattering cross sections, break up reaction [13–15] and spin observables [16–18]. Although, these studies show that by including the three-nucleon force the results of calculations of $p-d$ elastic scattering differential cross section were improved, a deviation from the experimental data is observed at backward scattering angles with increasing energy. Many authors of previous works attributed this to the absence of some short-range components of three-nucleon forces. But, we believe that at intermediate energy of the range, $100-300$ MeV, where most of the results were obtained, any very short-range components of three-nucleon force can be neglected. The same disagreement with experimental data at backward scattering angles can be expected for the results of multiple scattering model and OLA of Glauber. The reason is that these two approximations are obtained for small scattering angles at high energy where the backward scattering can

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be neglected. However, many authors used these approximations to calculate proton-nucleus and nucleus-nucleus reaction cross sections at low and intermediate energies [19,20].

Therefore, in this work we try to use the OLA to study p-d elastic scattering. At the same time, we will take the backward scattering and three-nucleon force effect into account. In Section 2, a short presentation of OLA is given, including three-nucleon force and the representation of backward scattering in elastic scattering amplitude is explained. The obtained results of p-d elastic scattering differential at the energies 108, 120, 135, 150, 170 and 190 MeV are discussed and given in Section 3, where the experimental data are taken from the Ref. [9]. The conclusions are given in Section 4.

2. A three-nucleon force approach in optical limit approximation: *p-d* scattering

We will briefly present the OLA including three-nucleon force, as given in Ref. [6] with a slight difference, where an interference term of two- and three-nucleon forces in the total nucleon-nucleon profile function is deleted. This deleted term was considered in Ref. [6], but, it is not physically accepted. The optical limit proton-deuteron (p-d) elastic scattering amplitude is given by

$$F_{opt}(\boldsymbol{q}) = \frac{ik}{2\pi}\int d\boldsymbol{b}\ \exp\{i\boldsymbol{q}.\boldsymbol{b}\}[1-\exp\{i\chi^{th}_{opt}(\boldsymbol{b})\}], \tag{1}$$

where $\hbar k$ is the incident momentum, $\boldsymbol{q}$ is the momentum transfer vector, $\boldsymbol{b}$ is the two- dimensional impact vector and the optical limit phase shift $\chi^{th}_{opt}(\boldsymbol{b})$, including the three-nucleon force effect, is

$$\chi^{th}_{opt}(\boldsymbol{b}) = \chi^{th}_{p}(\boldsymbol{b}) + \chi^{th}_{n}(\boldsymbol{b}), \tag{2}$$

where

$$\chi^{th}_{p}(\boldsymbol{b}) = \chi_{p}(\boldsymbol{b}) + \chi_{p;n}(\boldsymbol{b}), \tag{3}$$

$$\chi^{th}_{n}(\boldsymbol{b}) = \chi_{n}(\boldsymbol{b}) + \chi_{n;p}(\boldsymbol{b}), \tag{4}$$

where $\chi_{p(n)}(\boldsymbol{b})$is the two-nucleon force phase shift of scattering on target-proton (neutron), $\chi_{p;n}(\boldsymbol{b})$ and $\chi_{n;p}(\boldsymbol{b})$ are three-nucleon force corrections in each case. Using the general rule of OLA we get

$$\chi_{j}(\boldsymbol{b}) = i\iint d\boldsymbol{r}_p d\boldsymbol{r}_n \delta((\boldsymbol{r}_p+\boldsymbol{r}_n)/2))\ |\psi_d(\boldsymbol{r}_p,\boldsymbol{r}_n)|^2\Gamma_j(\boldsymbol{b},\ \boldsymbol{s}_j),\ \ j=p,\ n, \tag{5}$$

$$\chi_{j;k}(\boldsymbol{b}) = i\iint d\boldsymbol{r}_p d\boldsymbol{r}_n \delta((\boldsymbol{r}_p+\boldsymbol{r}_n)/2)\ |\psi_d(\boldsymbol{r}_p,\boldsymbol{r}_n)|^2\Gamma_{j;k}(\boldsymbol{b},\boldsymbol{s}_j;\boldsymbol{s}_k), \\ j,k=p,\ n;\quad j\neq k. \tag{6}$$

where ψ_d is the deuteron ground state wave function, the vectors $\boldsymbol{s}_j$ are the projections of the target nucleon position vectors $\boldsymbol{r}_j$ on the impact plane, $\Gamma_j(\boldsymbol{b}_j)$, $\boldsymbol{b}_j=\boldsymbol{b}-\boldsymbol{s}_j$, is the two-nucleon interaction profile function and $\Gamma_{j;k}(\boldsymbol{b}_j,\ \boldsymbol{s}_j;\boldsymbol{s}_k)\equiv \Gamma_{j;k}(\boldsymbol{b}_j,\ \boldsymbol{s}_{jk})$, $\boldsymbol{s}_{jk}=\boldsymbol{s}_j-\boldsymbol{s}_k$, is the correction due to the three-nucleon force, when the incident proton interacts with the *j*th target nucleon and, at the same time, the *k*th nucleon is considered as a spectator which interacts also with *j*th nucleon [5]. The Dirac $\delta-$ function expresses a constraint on the deuteron nucleus center of mass. Thus

$$\begin{aligned}\chi^{th}_{opt}(\boldsymbol{b}) &= \sum_{\substack{j,k=p,n\\ j\neq k}}\left[\chi_j(\boldsymbol{b})+\chi_{j;k}(\boldsymbol{b})\right] \\ &= i\iint d\boldsymbol{r}_p d\boldsymbol{r}_n\delta((\boldsymbol{r}_p+\boldsymbol{r}_n)/2))\ |\psi_d(\boldsymbol{r}_p,\boldsymbol{r}_n)|^2\sum_{\substack{j,k=p,n\\ j\neq k}}\left[\Gamma_j\big(\boldsymbol{b},\ \boldsymbol{s}_j\big)+\Gamma_{j;k}\big(\boldsymbol{b},\ \boldsymbol{s}_j;\ \boldsymbol{s}_k\big)\right].\end{aligned} \tag{7}$$

In this equation the interference term $-\Gamma_j(\boldsymbol{b},\ \boldsymbol{s}_j)\Gamma_{j;k}(\boldsymbol{b},\ \boldsymbol{s}_j;\ \boldsymbol{s}_k)$ of Ref. [6] is not considered. This term is not physically accepted, it means that the incident particle collides with the same target particle twice in the same scattering process.

The two-nucleon interaction profile function $\Gamma_j(\boldsymbol{b})$ is usually taken in the form

$$\Gamma_j(\boldsymbol{b}) = \frac{\sigma_j}{4\pi\beta_j^2}(1-i\varepsilon_j)\ \exp\left\{-\frac{b^2}{2\beta_j^2}\right\},\quad j=p,n \tag{8}$$

where σ_j is the nucleon-nucleon total cross section, ε_j is the ratio of real to imaginary parts of the elastic scattering amplitude in the forward direction and β_j is the slope parameter. The three-nucleon force profile function $\Gamma_{j;k}(\boldsymbol{b}_j,\ \boldsymbol{s}_{jk})$ will be