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Dual combination combination multi switching synchronization of eight chaotic systems

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ABSTRACT

In this paper, a novel scheme for synchronizing four drive and four response systems is proposed by the authors. The idea of multi switching and dual combination synchronization is extended to dual combination-combination multi switching synchronization involving eight chaotic systems and is a first of its kind. Due to the multiple combination of chaotic systems and multi switching the resultant dynamic behaviour is so complex that, in communication theory, transmission and security of the resultant signal is more effective. Using Lyapunov stability theory, sufficient conditions are achieved and suitable controllers are designed to realise the desired synchronization. Corresponding theoretical analysis is presented and numerical simulations performed to demonstrate the effectiveness of the proposed scheme.

1. Introduction

Much has been written and said about the concept of synchronization of chaotic systems since it was first introduced by Pecora and Caroll [1]. Because of its interdisciplinary nature the chaos synchronization problem has received interest from researchers across the academic fields such as physics, mathematics, engineering, biology, chemistry, etc. The potential applications of chaos synchronization to engineering systems, information processing, secure communications, and biomedical science amongst many others has led to a vast variety of research studies in this topic of nonlinear science [2–5]. Various kinds of synchronization such as complete synchronization, anti synchronization, projective synchronization, reduced order synchronization, etc. have been reported and presented in a variety of chaotic systems using many effective methods such as active control, adaptive control, backstepping control, sliding mode control and so on [6–13].

Amongst many synchronization schemes, dual synchronization of chaotic systems is one which has successfully piqued the scientific curiosity of researchers because of its challenging and non traditional nature. Deviating from the traditional approach of synchronizing one drive and one response system, in dual synchronization two drive systems are synchronized with two response systems. Since the first inception of the idea by Liu and Davids [14], it has been extensively investigated in various synchronization studies [15–17]. In all prior work the common theme is to consider one pair of drive system with

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one pair of response system. Only recently the idea of dual synchronization was extended to two pair of drive systems and one pair of response system [18,19].

Synchronization studies involving multiple drive and response systems is a relatively unexplored area of research. New ideas have recently been initiated in the study of chaos synchronization where multiple chaotic systems are involved. In the literature of chaos synchronization the addition of combination synchronization [20,21], combination combination synchronization [22,23], compound synchronization [24,25], double compound synchronization [26], compound combination synchronization [27,28] etc. has opened new research directions to be explored. These significant ideas have strengthened the security of information transmission because of the complexity which they bring in transmitted signals.

Multi switching synchronization of chaotic systems is yet another relatively unexplored area of research [29]. In this non conventional scheme, different states of the drive system are synchronized with different state of the response system. Due to this, a wide range of synchronization direction exists for multi switching synchronization schemes. The importance of such kind of studies to information security cannot be emphasised enough and thus makes them a very relevant topic to be investigated. A few reported work in this direction can be studied in [30–33]. To the knowledge of authors the diverse possibilities of multi switching synchronization have not yet been explored with the dual synchronization schemes.

In this paper, motivated by the above discussion, the authors have combined the idea of multi switching with dual synchronization and extended it to combination combination synchronization of four chaotic systems. The novel scheme, dual combination combination multi switching synchronization involves eight chaotic systems of which four are drive systems and four are response systems. In contrast to double compound synchronization involving four drive and two response systems, this work is a significant improvement and extension. Using Lyapunov stability theory, sufficient conditions have been achieved to realise the desired synchronization. To demonstrate the effectiveness of the proposed method numerical simulations have been performed. The main contribution and advantages of this study are: (a) The dual synchronization study has been extended to pair of two drive and two response systems to achieve dual combination combination synchronization. No such work involving two pair of response systems has earlier been reported. (b) The multi switching synchronization scheme is combined with dual combination combination synchronization to achieve the novel scheme which is a first of its kind. No previous work on dual multi switching studies exist. (c) The proposed scheme successfully synchronizes eight chaotic systems of which four are drive and four are response systems. The complexity of signals due to multiple combination and the number of synchronization directions due to multi switching vastly enhances the anti attack ability of any signal that will be transmitted using combination of two pair of drive systems. (d) Several existing synchronization schemes are obtained as special cases of dual combination combination multi switching synchronization.

2. Formulation of dual combination combination multi switching synchronization

In this section, we formulate the scheme of DCCMS of chaotic systems. We require two pair of four chaotic drive systems and two pair of four chaotic response systems. Let the first two drive systems be described as

$$\dot{x_1} = f_1(x_1) \tag{1}$$

$$\dot{x_2} = f_2(x_2) \tag{2}$$

where $x_1 = (x_{11}, x_{12}, \ldots, x_{1n})^T$, $x_2 = (x_{21}, x_{22}, \ldots, x_{2n})^T$, f_1, f_2: $R^n \to R^n$ are known continuous vector functions. Linear combination of the states of two drive systems (1) and (2) gives a resultant signal of the form

$$\begin{aligned} S_1 &= [a_{11}x_{11}, a_{12}x_{12}, \ldots, a_{1n}x_{1n}, a_{21}x_{21}, a_{22}x_{22}, \ldots, a_{2n}x_{2n}]^T \\ &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Ax \end{aligned} \tag{3}$$

where $A_1 = diag(a_{11}, a_{12}, \ldots, a_{1n})$, and $A_2 = diag(a_{21}, a_{22}, \ldots, a_{2n})$ are two known matrices and a_{1i}, a_{2j} are not all zero at the same time $(i, j = 1, 2, \ldots, n)$.

Next two drive systems are written as

$$\dot{y_1} = g_1(y_1) \tag{4}$$

$$\dot{y_2} = g_2(y_2) \tag{5}$$

where $y_1 = (y_{11}, y_{12}, \ldots, y_{1n})^T$, $y_2 = (y_{21}, y_{22}, \ldots, y_{2n})^T$, g_1, g_2: $R^n \to R^n$ are known continuous vector functions. Hence, the linear combination of the states of two drive systems (4) and (5) gives a resultant signal of the form

$$\begin{aligned} S_2 &= [b_{11}y_{11}, b_{12}y_{12}, \ldots, b_{1n}y_{1n}, b_{21}y_{21}, b_{22}y_{22}, \ldots, b_{2n}y_{2n}]^T \\ &= \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = By \end{aligned} \tag{6}$$

where $B_1 = diag(b_{11}, b_{12}, \ldots, b_{1n})$, and $B_2 = diag(b_{21}, b_{22}, \ldots, b_{2n})$ are two known matrices and b_{1i}, b_{2j} are not all zero at the same time $(i, j = 1, 2, \ldots, n)$.