Chinese Journal of Physics 55 (2017) 1917–1921

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.elsevier.com/locate/cjph

Radial perturbed magnetic field with different plasma pressures and viscosities

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ARTICLE INFO

Article history:
Received 27 January 2017
Revised 11 February 2017
Accepted 17 August 2017
Available online 21 August 2017

Key words:
Radial perturbed magnetic field
Plasma pressure
Viscosity

ABSTRACT

Evolution of the radial perturbed magnetic field with radius at different plasma pressures and viscosities is studied by using Fourier transforms and Finite difference method. The main conclusions are that the effect of lower plasma pressure on radial perturbed magnetic fields is not evident. Higher plasma pressure has obvious effect on longer waves; however, shorter waves were affected by lower plasma pressure obviously. The maximum values of radial perturbed magnetic fields increase with the increase of plasma viscosities which indicates plasma viscosity cannot be neglected in the precise calculation.

1. Introduction

It is well known that magnetic field and plasma viscosity ν are the most important factors in the study of magnetohydrodynamics (MHD) in many fields. For example, Electron heat diffusion is affected by perturbed magnetic field in Tokomak [1]. The power-law distributions suggest the existence of nontrivial build-up and relaxation mechanisms for the magnetic energy in the solar atmosphere [2]. Liani and Ardjani studied the magnetic field effect on the electrons' thermal equilibrium using the Monte Carlo method and obtain the conclusion that the steady state of the electrons' energy is always achieved if the time is long enough [3]. The effect of external magnetic field on the properties of the higher-order electron-acoustic solitary waves is significant [4]. Chuang et al. studied the characteristics of hydromagnetic waves in dusty plasmas; the conclusions indicated that it is possible to have whistler and cyclotron waves with left-handed and right-handed magnetic polarizations due to the presence of massive charged dust grains [5].

On the other hand, the plasma viscosity will cause the variation of magnetic field and displacement in MHD. For instance, the shear viscosity rapidly increases near the deconfinement transition in a phenomenological model analogous to the quark gluon plasma [6]. Shear viscosity has evident effect on electric conductivity for the quark gluon plasma [7]. Rose et al. investigate the nonzero bulk viscosity coefficient on the azimuthal momentum at the Large Hadron Collider and get the conclusion that a finite bulk viscosity coefficient leads to a better description of the flow harmonics in ultracentral collisions [8]. The effect of the plasma viscosity on pure elements and multicomponent mixtures can predict the appearance and the growth of hydrodynamic instabilities in the inertial confinement fusion [9]. Effect of shear viscosity on the weak and strong coupling quasi gluon plasma demonstrate the viscosity ratio is uniquely determined by a quadratic dependence [10].

There are many means to study the perturbed parameters in MHD. The perturbed field of the International Thermonuclear Experimental Reactor is calculated with numerical calculation method [11]. Analysis of the behavior and relevant

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http://dx.doi.org/10.1016/j.cjph.2017.08.019
0577-9073/

parameters for the electron and ion plasma temperatures in the ionosphere is briefly discussed with the scatter radar data [12]. A semianalytical method for studying linear instability of the kink mode in a cylindrical plasma with line-tied boundary conditions is introduced by Evstatiev [13]. Dai and Wang discussed the radial perturbed displacement in a cylindrical plasma column and presented the relationship between the plasma pressure and the radial perturbed displacement. However, the plasma viscosity is not mentioned in this paper [14].

The main purpose of this paper is to calculate the effect of plasma pressure and viscosity on the radial perturbed magnetic field.

2. Model of the MHD equations

Non-ideal MHD equations which contain viscosity item can be written as [15, 16]

$$\rho_0 \frac{\partial \mathbf{u}_1}{\partial t} = -\nabla P_1 + (\nabla \times \mathbf{B}_0) \times \mathbf{B}_1 + (\nabla \times \mathbf{B}_1) \times \mathbf{B}_0 + \nu \rho_0 \nabla^2 \mathbf{u}_1 \tag{1}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{u}_1 \times \mathbf{B}_0) \tag{2}$$

$$\frac{\partial \rho_1}{\partial t} + \nabla \cdot (\rho_0 \mathbf{u}_1) = 0 \tag{3}$$

$$\nabla \cdot \mathbf{B}_1 = 0 \tag{4}$$

$$\frac{\partial}{\partial r}\left(P_0 + \frac{B_{0\theta}^2}{2} + \frac{B_{0z}^2}{2}\right) + \frac{B_{0\theta}^2}{r} = 0 \tag{5}$$

Eq. (5) is pressure balance equation. A zero subscript indicates equilibrium quantities, and a one subscript indicates perturbed quantities in Eqs. (1)–(5). $\mathbf{B}_0$ is the initial magnetic field and satisfy with the relationship $|\mathbf{B}_0|^2 = B_{0\theta}^2 + B_{0z}^2$, $B_{0\theta}$ is initial azimuthal magnetic field, B_{0z} is initial axial magnetic field,P_0 is initial plasma pressure, ν is the plasma viscosity,ρ is plasma density. Expansion form of Fourier transforms for perturbed parameters in Eq. (1)–(4) is $\exp[i(m\theta + kz - \omega t)]$. For convenience of calculation, we let the plasma displacement ξ, then

$$\mathbf{u} = \mathbf{u}_1 = \frac{d\mathbf{r}}{dt} = \frac{\partial \xi}{\partial t} + \mathbf{u}_1 \cdot \nabla \xi \approx \frac{\partial \xi}{\partial t} \tag{6}$$

The second-order ordinary differential equation and radial perturbed magnetic field $\mathbf{B}_{1r}$ are calculated by using Fourier transforms.

$$\frac{d}{dr}\left[\frac{D}{C_2}\frac{1}{r}\frac{d}{dr}(r\xi_r)\right] + \left[\frac{1}{D}\left(C_3 - \frac{C_1^2}{C_2}\right) - r\frac{d}{dr}\left(\frac{C_1}{rC_2}\right)\right]\xi_r = 0 \tag{7}$$

$$\mathbf{B}_{1r} = iF\xi_r - \frac{B_{0\theta}\xi_\theta}{r} + \frac{\xi_\theta B_{0\theta}}{r} \tag{8}$$

where $F = mB_{0\theta}/r + kB_{0z}$, k is wave number, m is the azimuthal module, we let $m = 1$ in this paper. coefficients D, C_1, C_2 and C_3 in Eq. (7) are presented in Dai and Wang [14]. The growth rate $\gamma = -i\omega$ is defined in the following calculations. The growth rate can be obtained by calculating Eq. (7) when the $B_{0\theta}$,B_{0z} and P_0 are fixed. Then the radial perturbed magnetic fields with a fixed growth rate are calculated.

3. Results and discussion

The initial magnetic fields $B_{0\theta}, B_{0z}$ and initial plasma pressure P_0 should be given before calculation. The corresponding expressions can be written as [14]

$$B_{0\theta} = \frac{r}{1+r^2}, B_{0z} = \frac{c}{1+r^2}, P_0 = \frac{(1-c^2)}{2(1+r^2)^2} \tag{9}$$

where c is a parameter to adjust the initial magnetic fields $B_{0\theta}, B_{0z}$ and initial plasma pressure P_0. The formulas in Eq. (9) satisfy the pressure balance equation in Eq. (5), so the expressions of $B_{0\theta}$, B_{0z} and P_0 are reasonable. Two cases of the parameter c ($c = 0.98$ and $c = 0.9$) are considered in this paper. The relationship between wave number k and growth rate γ with different parameter c is shown in Fig. 1 The dash lines and the solid lines are correspond to $c = 0.9$ and $c = 0.98$, respectively. The initial magnetic fields $B_{0\theta}, B_{0z}$ and initial plasma pressure P_0 can be calculated when the parameter c is fixed. For convenience of calculation and comparison, we choose a stationary growth rate $\gamma = 0.03008$ in this paper. When the range of growth rate γ and wave number k with different plasma pressure has been determined in Fig. 1, now we can give the evolution of radial perturbed magnetic $\mathbf{B}_{1r}$ with radius at different wave numbers k and plasma viscosities ν.