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Frequency response of the decoherence in a Duffing oscillator and the dispersion of the Wigner function in the Fourier domain

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ABSTRACT

The decoherence measured by the linear entropy is shown to be closely related to the dispersion of the Wigner function in the Fourier domain. The latter corresponds to the oscillations of the Wigner function in the phase space and can be influenced by the external driving force in driven nonlinear systems, e.g., in a Duffing-type oscillator. In the further investigations on the Duffing-type oscillator, the growth of the entropy of the system is found to be significantly dependent on the frequency of the driving force. Pronounced response peak occurs in the driving frequency response curve of the entropy of the system. Furthermore, there is good correspondence between the frequency response curve of the entropy and that of the dispersion of the Wigner function in the Fourier domain.

1. Introduction

The environment-induced decoherence is a central concept in physics. It is viewed as a major obstacle for realizing the applications of quantum coherence in the fields such as quantum computer and quantum simulation. It has received much attention for many years and still of great interest [1–7]. Early studies have shown that decoherence is closely related to chaos [4–7]. Besides, it is also found that universal decoherence exists in systems with gravitational potential [8] and macroscopic quantum resonators [9]. During recent years, the experimental tests of quantum phenomena in mesoscopic and macroscopic objects provide new and novel insights into the fundamental questions in quantum physics involving decoherence [9]. In the studies on the quantum phenomena in mesoscopic and macroscopic systems, driven nonlinear oscillators are of fundamental interests [10–20]. Known to all, frequency response is one of the important features in driven nonlinear systems. It is widely used in information technologies and also can be found in many studies on mesoscopic and macroscopic systems [10–20]. Due to the coupling to the environment, the driven nonlinear systems mentioned above are often subjected to the influence of the environment. The latter tends to lead to decoherence. However, the frequency response of the decoherence in driven nonlinear systems has not been clarified.

In this paper, the relations of the decoherence to the dispersion of the Wigner function in the Fourier domain and the frequency response of the decoherence are investigated in an open driving oscillator by means of the Wigner function. The decoherence measured by the linear entropy is shown to arise from the loss of the high frequency components of

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the Wigner function in the Fourier domain. Its growth rate increases with the increase of the dispersion of the Wigner function in the Fourier domain. The latter corresponds to the increase of the oscillations of the Wigner function in the phase space. It can be influenced by the driving force and shown driving frequency dependence in a periodically driven nonlinear system, e.g., in a Duffing-type oscillator. In the investigations on the Duffing-type oscillator, the average growth rate of the entropy of the system is found to significantly depend on the frequency of the external driving force. It increases with increasing the driving frequency from zero until a maximum is reached, after which it rapidly drops off to a plateau. Accordingly, a pronounced peak can be seen in the frequency response curve of the entropy of the system. Furthermore, good correspondence is found between the frequency response curves of the entropy and the dispersion of the Wigner function in the Fourier domain.

2. Decoherence and the dispersion of the Wigner function in the Fourier domain

The model studied in this work is a periodically driven nonlinear system, which is of fundamental and technical interests in the studies related to decoherence and mesoscopic devices [4–6,10–13,18–22]. Its Hamiltonian can be written as

$$H_S = \frac{p^2}{2m} + V(q,t), \tag{1}$$

where $V(q,t) = V_0(q) + f_0 q\cos(\omega t)$. $V_0(q)$ is the potential of the nonlinear oscillator and is responsible for the external driving force.

The quantum state of the system (1) is described by the Wigner function [23]. The latter is the Wigner–Weyl transform of the density matrix and can be written as

$$W(q,p;t) = (2\pi\hbar)^{-1}\int dy e^{ipy/\hbar}\left\langle q-\frac{y}{2}\middle|\rho(t)\middle|q+\frac{y}{2}\right\rangle \tag{2}$$

It was introduced to study the quantum corrections to classical statistical mechanics and is the quantum analogue of the classical phase-space distribution [23]. The Wigner function is of fundamental importance in many fields of quantum physics, e.g., in quantum computation [24] and in quantum optics [25]. Besides, it can be measured directly via homodyne tomography [25] and its oscillations in phase space are signature of the quantum interference [25–27].

The coupling of the system (1) to environment is modeled via the Ohmic environment in high-temperature and weak-coupling limit. In this case, the time evolution of the Wigner function of the system (1) can be described as [4,28,29]

$$\partial_t W = \{H_S, W\}_{PB} + \sum_{n\geq 1}\frac{(-1)^n(\hbar/2)^{2n}}{(2n+1)!}\partial_q^{2n+1}V\partial_p^{2n+1}W + D\partial_p^2 W, \tag{3}$$

where $\{\}_{PB}$ is the Poisson bracket, the last term on the right hand side arises from the coupling of the system to the environment and D is the so-called diffusion coefficient.

In the Wigner representation, the decoherence of the system (1) can be evaluated by the linear entropy [30,31]

$$S(t) = 1 - 2\pi\hbar\int dq\int dpW^2(q,p;t). \tag{4}$$

It can be expressed in terms of the reduced density of the system $S(t) = 1 - Tr\rho^2$ $(0 \leq S \leq 1)$. Its growth indicates the decay of the off-diagonal elements of the reduced density matrix of the system and thus corresponds to the loss of the quantum coherence [30]. In other words, the decoherence increases as S increases from 0 to 1.

To investigate the environment-induced decoherence, it is necessary to explore the evolution of the Wigner function of the system. The latter is described by Eq. (3). Its solution in the Heisenberg picture is

$$W(q,p;t) = \exp\left[h_0 t + h_E t + F(t)\right]W(q,p;0), \tag{5}$$

where $h_0 = -(p/2m)\partial_x + (\partial_q V)\partial_p + \left[(-\hbar^2)^n 2^{-2n}/(2n+1)!\right]\partial_x^{2n+1}V\partial_p^{2n+1}$, $h_E = D\partial_p^2$ and $F(t) = (f_0/\omega)\sin(\omega t)$. For a short time interval δt, Eq. (5) can be approximated as

$$W(q,p;t+\delta t) = U_D(\delta t)W_0(q,p;t+\delta t) + O(\delta t^2), \tag{6}$$

where $U_D(\delta t) = \exp(h_E\delta t)$ and $W_0(q,p;t+\delta t) = \exp[h_0\delta t + F(t)]W(q,p;t)$. $W_0(q,p;t+\delta t)$ corresponds to the evolution in the absence of diffusion.

By taking the Fourier transform with respect to p, Eq. (6) becomes

$$\widetilde{W}(q,p_f;t+\delta t) \simeq e^{-D\delta t p_f^2}\,\widetilde{W}_0(q,p_f;t+\delta t), \tag{7}$$

where $\widetilde{W}(q,p_f;t) = \int dpe^{-ip_f p}W(q,p;t)$ is the Fourier transform of $W(q,p;t)$ with respect to p. In Eq. (7), the exponential decay factor before $\widetilde{W}_0$ arises from $U_E(\delta t)$ and is caused by the coupling to the environment. Its exponent increases with $|p_f|$ and thus it induces the decay of the oscillations of the Wigner function in the phase space via cutting off the high-frequency components of $\widetilde{W}_0$ in the Fourier domain. This can lead to decoherence, since the spatial oscillations of the Wigner function are signature of the quantum interference and indicates the coherence of the quantum state of the system.