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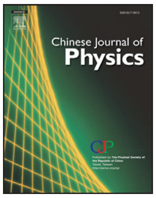

Effects of relativistic electrons and spatial non-uniformities of density on periodic absolute parametric instability in a plasma waveguide

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A B S T R A C T

The effects of relativistic electrons and spatial plasma nonuniformity of density on periodic absolute parametric instability (API) of electrostatic waves in a relativistic $1-D$ nonuniform magnetoactive plasma are investigated in a cylindrical geometry. We use the separation method to solve the two-fluid plasma equations, which describe the system. The method used enables us to determine the frequencies and growth rates of unstable modes and the self-consistent electric field. Plasma electrons are considered to have a relativistic velocity. Increment has found in the buildup of the oscillations, and it is shown that the spatial nonuniformity of the plasma exerts a stabilizing effect on the parametric instability. It is shown that the growth rate of the API in relativistic plasma is reduced compared to nonrelativistic plasma. Independent of the geometry of the problem (plane and cylinder), the results of the API in a relativistic plasma waveguide are still valid.

The proposal approach (ʒseparation methodȝ) is significantly simpler that the method ordinarily employed in the theory of parametric resonance in nonuniform plasma. Therefore, it is of special interest to apply the separation method to solution of different problems involving parametric excitation of electrostatic waves in bounded nonuniform plasma.

1. Introduction

The absolute parametric instability (API) of the low-frequency oscillations excited by a monochromatic pumping field of an arbitrary amplitude in a cold nonuniform magnetoactive plasma is investigated by Silin, V. P. (1965) [1]. Increments have found in the buildup of the oscillations, and it has shown that the spatial nonuniformity of the plasma exerts a stabilizing effect on the parametric instability. An API plays a crucial role in the processes of the energy transfer from the electromagnetic radiation to the plasma and may have important consequences for experiments on RF plasma heating in magnetic traps and for a laser-fusion system.

A great deal of attention has been devoted to the investigation of absolute parametric instability in nonuniform plasma. Parametric instability of the decay type excited by a monochromatic pumping field of small amplitude in isotropic plasma has been discussed in [2–14]. They are shown that a high frequency electric field has no essential influence on dispersion

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characteristics of unstable surface waves excited in a plasma waveguide by a low-density electron beam. The region of instability is only slightly narrowing and the growth rate decreases by a small parameter and this result was reduced comparing to cold, non relativistic plasma. Also, it isfound that the plasma electrons did not affect the solution of the space part of the problem.

For investigation of a parametric effects in a uniform plasma under the action of an intense HF (pump) electric field the method developed in refs. [1,15] is commonly used. This method was modified [16] in order to investigate the API in a magnetized nonuniform plasma by a HF electric field of an arbitrary amplitude. Demchenko et al. (1976) [17] have reported an analysis of the effect of spatial plasma nonuniformity on parametric instability of electrostatic waves in a magnetized cylindrical waveguides subjected to an intense HF electric field.

Many authors had investigated the density bunch formation by microwave in a plasma-filled cylindrical [18,19] and rectangular [20–23] waveguide. These bunches can be tailored uniformly by varying the microwave frequency, microwave intensity, electron temperature and initial plasma density profile in the waveguide. The effect of the microwave frequency and width of the waveguide is to decrease to the electron energy gain for a fixed microwave intensity. However, higher acceleration gradients are obtained when the electron is injected with a large energy.

It is already known that, nonuniformity of the plasma density leads to: (i) an increase of the threshold value of the pump wave amplitude above which parametric amplification occur and (ii) the localization of an unstable waves in a finite region of a plasma. This suggests that instability has assumed to have an absolute character. It should be emphasized that, from an experimental point of view, it is quite important to know whether a given parametric instability is absolute or convective. This is so essential because the nature of the parametric instability determines the mechanism of their saturation. The convective instability reaches saturation at a comparatively low level, due to the convection of energy of the decay product (secondary waves) away from the three-wave resonance region. The API saturates at a higher level under the action of various nonlinear effects. From this point of view, an API plays a crucial role in the process of the energy transfer from the electromagnetic radiation to the plasma and may have important consequences for experiments of RF plasma heating in tokamaks and for a laser fusion.

A method is expounded in a paper that permits reducing the problem of absolute parametric instability excited by a monochromatic pumping field of arbitrary amplitude in nonuniformmagnetoactive plasma to the problem of parametric excitation of spatial oscillations in uniform isotropic relativistic plasma. Below we will discuss the parametric excitation of low-frequency waves whose dispersion is completely determined by a high-frequency field, in a strong magnetic field i.e., here, we shall apply the method of ref. [17] for investigate of the API in a $1-D$ nonuniform relativistic plasma waveguide subjected to an intense HF electric field as an eigen-value problem.

Here we shall study the method of refs. [6,7,9–17] for investigate of the API in $1-D$ nonuniform bounded relativistic cylindrical plasma waveguide.

2. Separation method in the problem of API in a $1-D$ nonuniform bounded relativistic plasma

Let us consider cylindrical waveguide is filled by nonuniform bounded relativistic plasma with parabolic radial density distribution $(n_{\alpha_0} = n_{\alpha_0}(r) = N(1 - r^2/R^2);\ \alpha = e, i)$ (where $e$and i are denotes to electron and ion plasma). The radius of the beam R is supposed to coincides with the radius of the plasma cylinder$r$. A uniform strong static magnetic field $\vec{B}_0\,(\omega_{c_\alpha} \gg \omega_{P_\alpha})$ (where ω_{c_α} and ω_{P_α} are the cyclotron and plasma frequencies for sort α respectively) and a HF electric field $\vec{E}_P = \vec{E}_0 \sin(\omega_0 t)$ are directed along the z axis.We choose the electric field of an ordinary wave (sin wave) as an HF pump field.

The initial system of equations consists of the two fluid equations in combination of the Poisson equation:

$$\frac{\partial(m\vec{V}_\alpha)}{\partial t} + \left(\vec{V}_\alpha \cdot \nabla\right)\left(m_\alpha \cdot \vec{V}_\alpha\right) = e_\varepsilon\left(\vec{E}_P + \frac{1}{C}\left[\vec{V}_\alpha \times \vec{B}_0\right] - \nabla\Phi\right), \tag{1}$$

$$\frac{\partial n_\alpha}{\partial t} + div\left(n_\alpha \vec{V}_\alpha\right) = 0 \tag{2}$$

$$\Delta\Phi = -4\pi \sum_\alpha e_\alpha n_\alpha \tag{3}$$

where, n_α and $\vec{V}_\alpha$ are the density and velocity of particles of species α, and Φ is the potential self-consistent electric field.

The equilibrium particles velocity $\vec{u}_\alpha(0,\ 0,\ u_\alpha)$ is determined by the following expression:

$$\vec{u}_\alpha = -\frac{e_\alpha\,\gamma\,\vec{E}_0}{m_\alpha\omega_0}\cos(\omega_0 t) \tag{4}$$

Representing the perturbations of velocity $\delta\vec{V}_\alpha(0,\ 0,\ \delta V_\alpha)$, density δn_α and electrical potential Φ in the form $(\delta\vec{V}_\alpha,\ \delta n_\alpha,\ \Phi) \sim \exp i(m\Psi + kz)$. It means that particles are ʒfrozenʒ and could not move across the magnetic field $\delta V_{\alpha_r} = \delta V_{\alpha_\Psi} = 0$.

Suppose that $k_z \equiv k;\quad \frac{\partial\delta V_\alpha}{\partial t} \sim \omega\delta V_\alpha \sim \omega_{P_\alpha}\delta V_\alpha,\ ku_\alpha \sim \omega_{P_\alpha}$.