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Bayesian Compressive Sensing for Thermal Imagery Using Gaussian-Jeffreys Prior

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Abstract

Recent advances have shown a great potential to explore compressive sensing (CS) theory for thermal imaging due to the capability of recovering high-resolution information from low-resolution measurements. In this paper, we present a Bayesian CS reconstruction algorithm that makes use of a new sparsity-inducing prior, referred as Gaussian-Jeffreys prior, and demonstrate performance gain of imposing this new prior on thermal imagery where the signal-to-noise ratio is low. We first derive a hierarchical representation of the Gaussian-Jeffreys prior that facilitates computational tractability, then propose an efficient evidence approximation inference algorithm. We show that the proposed estimator is able to provide stronger sparsity-inducing power comparing to the conventional choices. Extensive numerical examples are provided with performance comparisons of different CS estimators, in particular when the compressive measurements are available via thermal imaging.

Keywords: sparse estimation, Gaussian-Jeffreys prior, Bayesian modeling, thermal imagery, noisy measurements

1. Introduction

Compressive sensing (CS) theory [1, 2, 3] proposes new techniques to recover unknown sparse signals from underdetermined linear measurements, making the use of the sensors with much fewer detectors possible. As for infrared imagers the sensor resolution has a significant impact on the hardware cost, CS has become one of the most compelling research topics in this area. Recent advances includes single-pixel infrared detector[4, 5, 6], super-resolution reconstruction[7] and image fusion [8, 9].

The CS theory indicates that, if a signal is compressible in the basis, then highly accuracy recovery is possible with incomplete measurements. Specifically, let $\mathbf{u} \in \mathbb{R}^M$ represent original signal, $\mathbf{u}$ is compressible in basis $\mathbf{\Psi} \in \mathbb{R}^{M\times M}$ (e.g. a wavelet basis), which means that $\mathbf{u} = \mathbf{\Psi w}$ and most components of the vector $\mathbf{w} \in \mathbb{R}^M$ have negligible amplitude. Write $\mathbf{w}$ as $\mathbf{w} = \mathbf{x} + o(\mathbf{x})$, where $\mathbf{x} \in \mathbb{R}^M$ represents $\mathbf{w}$ with all the negligible components set to zero. According to CS theory, $\mathbf{x}$ is guaranteed to be recovered from an insufficient number of measurements, denoted by $\mathbf{y} \in \mathbb{R}^N$, $\mathbf{y} = \mathbf{\Phi\Psi}^T\mathbf{u} = \mathbf{\Phi w} = \mathbf{\Phi x} + \mathbf{\Phi}o(\mathbf{x}) = \mathbf{\Phi x} + \mathbf{e}_o$, where $N \ll M$, $\mathbf{e}_o = \mathbf{\Phi}o(\mathbf{x})$, $\mathbf{\Phi} \in \mathbb{R}^{N\times M}$ is measurement matrix. Also note that the measurements may be noisy, with measurement noise $\mathbf{e}_m$, thus $\mathbf{y} = \mathbf{\Phi x} + \mathbf{e}_o + \mathbf{e}_m$. Let $\mathbf{e} = \mathbf{e}_o + \mathbf{e}_m$, represented by a zero-mean white Gaussian noise, finally, the CS model is given by:

$$\mathbf{y} = \mathbf{\Phi x} + \mathbf{e}. \tag{1}$$

Eqn.(1) is underdetermined, since the number of measurements $\mathbf{y}$ is much smaller then the number of coefficients $\mathbf{x}$. Solving for it requires a sparsity-promoting regularization on $\mathbf{x}$. Essentially, most successful current CS reconstruction algorithms [2, 10, 11, 12, 13, 14] perform least squares regression with the addition of independent generalized Gaussian prior over each coefficient:

$$\mathrm{p}(x) \propto \exp\left(-|x|^p\right), \tag{2}$$

when $p \in [0, 1]$, $\exp(-|x|^p)$ has been proved to encourage sparsity due to its heavy tails and sharp peak at zero. Specifically, when $p = 1$, exponent of the multi-dimensional prior $\mathrm{p}(\mathbf{x})$ is an ℓ_1-norm; when $p \to 0$, exponent of $\mathrm{p}(\mathbf{x})$ approaches an ℓ_0-norm, i.e. a count of the number of nonzero components in vector $\mathbf{x}$, defined as $\|\mathbf{x}\|_0 \triangleq \sum_m 1_{\{x_m \neq 0\}}$. Accordingly, the *Maximum a posteriori* (MAP) estimation using $\mathrm{p}(\mathbf{x})$ is a penalized least squares regression with ℓ_p-norm:

$$\arg\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{\Phi x}\|_2^2 + \eta\|\mathbf{x}\|_p, \tag{3}$$

where model parameter η controls the relative importance applied to error term and sparseness term. A number of methods have been proposed to solve the MAP problem defined in Eqn.(3), including linear programming algorithms [2, 10], reweighted norm algorithms [11, 12] and greedy methods [13, 14].

There is also a significant trend to formulate the CS reconstruction problem in a Bayesian framework, which